

# A two-class fiscal and monetary growth model

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## Abstract

This paper extends the two-class monetary growth model by incorporating fiscal activity along the lines of Pasinetti (Cambridge J. Econ. 13, 25–36). Despite this extension, the Cambridge equation is found to be one of long-run equilibria. It is shown that Fiscal policy decreases the accumulation of capital, through an increase in the proportional tax on profits. An inflationary monetary policy has ambiguous effects, it can have positive, negative or no effect on the accumulation of capital and on the rate of profit. © 2000 Elsevier Science B.V. All rights reserved.

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## 1. Introduction

This paper extends Ramanathan's (1976) two-class monetary growth model by incorporating fiscal activity along the lines of Pasinetti (1989). Following the Samuelson and Modigliani (1966) approach, the Kaldor-Pasinetti growth model is blended with some neoclassical assumptions<sup>1</sup>. The two-class monetary growth model is rewritten as a system of differential equations. The Cambridge equation, also known as the Pasinetti paradox, is one of the possible steady-state equilibria of this system, despite the inclusion of monetary and fiscal variables. This is a

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<sup>1</sup> See also Darity (1981), O'Connell (1995) and Faria and Teixeira (1999).

remarkable result, because workers' demand for money and the taxes on their income do not affect the long-run growth of the economy.

The study of the Cambridge equation equilibrium permits us to analyse the impact of monetary and fiscal policies on long-run growth. It is shown that taxation on profits is positively related to the rate of profit, decreasing the accumulation of capital. An inflationary monetary policy is found to have ambiguous effects on growth, and its power depends on the capitalists demand for money and its sensitivity to inflation.

The paper is organised as follows. The next section presents Pasinetti's model with fiscal activity. Section 3 introduces this model into the framework adapted from Ramanathan (1976). The model is rewritten as a dynamic system, and one of the steady-state equilibria, the Cambridge equation, is studied. The impact of inflation and taxation is analysed. Finally, Section 4 concludes.

## 2. Fiscal activity

Pasinetti (1989) has studied the role of government taxation and expenditure in Kaldorian-type models of growth and distribution. He has explored alternative budget constraints (equilibrium, surplus and deficit) in a model in which both direct and indirect taxes are considered.

National income ( $Y$ ) is divided into consumption ( $C$ ), autonomous investment ( $I$ ), and government expenditure ( $G$ ):

$$Y \equiv C + I + G \equiv W + P \quad (2.1)$$

where the right hand side represents the income distribution between wages ( $W$ ) and profits ( $P$ ). Total profits are divided between profits accruing to workers and capitalists ( $P_W$ ,  $P_C$ ), respectively:

$$P = P_W + P_C \quad (2.2)$$

The government expenditures is related to total taxation ( $T$ ) as:

$$G = (1 - s_G)T \quad (2.3)$$

where  $s_G$  is the propensity to save of the government. The government total revenue from taxation ( $T$ ) depend on the taxation of profits, wages and on all consumption expenditure of individuals and the government itself:

$$T = t_W W + t_P (P_W + P_C) + t_i \{ (1 - s_W) [(1 - t_W) W + (1 - t_P) P_W] + (1 - s_C) (1 - t_P) P_C + G \} \quad (2.4)$$

where  $t_W$ ,  $t_P$ ,  $t_i$  denote, respectively, the proportional direct tax on wages, profits and all consumption expenditure.

The saving functions for the various categories of savers are the following:

$$S = S_W + S_C + S_g \quad (2.5)$$

$$S_W = s_W [(1 - t_W) W + (1 - t_P) P_W] \quad (2.6)$$

$$S_C = s_C(1 - t_P)P_C \tag{2.7}$$

$$S_G = s_G T \tag{2.8}$$

where the  $S$  stands for total savings,  $s$  denotes the marginal propensity to save, and the subscripts  $W$ ,  $C$  and  $G$ , correspond to workers, capitalists and government.

The usual assumptions are:  $0 < t_W < t_P < 1$ ,  $0 \leq s_W < s_C < 1$ .

Substituting Eq. (2.3) into Eq. (2.4) yields the total taxation function in Eq. (2.9), and introducing Eq. (2.9) into Eq. (2.8) yields the government saving function in Eq. (2.10):

$$T = \chi \{ t_W W + t_P P_W + t_P P_C + t_i [(1 - s_W)(1 - t_W)W + (1 - s_W)(1 - t_P)P_W + (1 - s_C)(1 - t_P)P_C] \} \tag{2.9}$$

$$S_G = s_G \chi \{ t_W W + t_P P_W + t_P P_C + t_i [(1 - s_W)(1 - t_W)W + (1 - s_W)(1 - t_P)P_W + (1 - s_C)(1 - t_P)P_C] \} \tag{2.10}$$

where  $\chi = [1 - t_i(1 - s_G)]^{-1}$  is the correction factor as a result of the fact that the government also taxes its own expenditure<sup>2</sup>.

Substituting Eqs. (2.6), (2.7) and (2.10) into Eq. (2.5) we have:

$$S = s'_{WW}W + s'_{WC}P_W + s'_C P_C \tag{2.11}$$

where

$$s'_{WW} = s_W(1 - t_W) + s_G \chi [t_W + t_i(1 - s_W)(1 - t_W)] \tag{2.12}$$

$$s'_{WC} = s_W(1 - t_P) + s_G \chi [t_P + t_i(1 - s_W)(1 - t_P)] \tag{2.13}$$

$$s'_C = s_C(1 - t_P) + s_G \chi [t_P + t_i(1 - s_G)(1 - t_P)] \tag{2.14}$$

The standard Pasinetti's assumptions are: (i) savings are proportional to capital for both capitalists and non-capitalists; (ii) the investments equal savings (ex-ante equilibrium condition); and finally (iii) there is an uniform equilibrium rate of profit for the economy as a whole. Combining these assumptions with the equations above we can derive the extended Cambridge equation,  $r = g/s'_C$ , as shown by Pasinetti (1989). This result extends the Pasinetti paradox for models with fiscal activity. It means that workers' propensity to save does not play any role in determining the rate of profit of the economy.

### 3. The two-class fiscal and monetary growth model

In order to deal with the monetary model we follow a version of Ramanathan (1976), with the provision that now fiscal policy is considered. In this sense we extend the two-class monetary growth model including the public sector. The model is simplified into a system of two differential equations. One for the capital stock

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<sup>2</sup> This is not a fundamental assumption, notice that if  $t_i = 0$  then  $\beta = 1$ , which does not affect any of our further results.

owned by capitalists and the other for the total physical capital. The Cambridge equation emerges as one of the possible steady-state solutions of this system.

There are two stores of value, physical capital ( $K$ ), and currency ( $M$ ). The stock of currency, can be varied only by budget deficits or surpluses:

$$G - T = \frac{\dot{M}}{p} \quad (3.1)$$

where  $p$  is the price of goods in terms of currency. Eq. (3.1) replaces Eq. (2.3). So there is no government marginal propensity to save in the model that follows. This implies that equations Eqs. (2.12), (2.13) and (2.14) are simplified to:

$$s'_{WW} = s_W(1 - t_W) \quad (2.12')$$

$$s'_{WC} = s_W(1 - t_P) \quad (2.13')$$

$$s'_C = s_C(1 - t_P) \quad (2.14')$$

Total real wealth ( $R$ ) at any moment of time is split between capital and real money balances ( $m = M/p$ ) (see Tobin, 1955):

$$R = K + \frac{M}{p}$$

Total savings are allocated to investment in physical capital ( $\dot{K}$ ) and in real money balances ( $\dot{m}$ ):

$$\dot{K} + \dot{m} = S - \pi m \quad (3.2)$$

This is the budget constraint of the economy<sup>3</sup>.

Substituting Eq. (2.11) into Eq. (3.2) we obtain:

$$\begin{aligned} S - \pi m &= \dot{K} + \dot{m} \Rightarrow \dot{K}_W + \dot{K}_C + \dot{m}_C + \dot{m}_W \\ &= s'_{WW}W + s'_{WC}P_W + s'_C P_C - \pi(m_C + m_W) \end{aligned} \quad (3.3)$$

Note that the marginal saving propensities are given now by Eqs. (2.12'), (2.13') and (2.14').

From Eq. (3.3) we have:

$$\dot{K}_W = s'_{WW}W + s'_{WC}P_W - \dot{m}_W - \pi m_W \quad (3.4)$$

$$\dot{K}_C = s'_C P_C - \dot{m}_C - \pi m_C \quad (3.5)$$

One can define the following terms (see list of variables in Appendix 1):

$$\dot{L}/L = g \quad (3.6)$$

$$k = K/L \quad (3.7)$$

$$K = K_C + K_W \Rightarrow k_C + k_W = 1 \quad (3.8)$$

$$\dot{k}_C = -\dot{k}_W \quad (3.9)$$

<sup>3</sup> See also Sidrauski (1967).

$$z = M_C/M \tag{3.10}$$

$$\mu = \dot{M}/M \tag{3.11}$$

$$\pi = \dot{p}/p \tag{3.12}$$

$$\frac{\dot{M}_C}{pK_C} = \frac{m_C}{K_C} = \delta(k, \pi), \quad \delta_k > 0, \quad \delta_\pi < 0 \tag{3.13}$$

$$\frac{\dot{M}_W}{pK_W} = \frac{m_W}{K_W} = \beta(k, \pi), \quad \beta_k > 0, \quad \beta_\pi < 0 \tag{3.14}$$

Where Eq. (3.13) denotes the capitalists demand for money and Eq. (3.14) stands for the workers demand for money.

Now we are able to derive the dynamic system for total physical capital, and capital stock owned by capitalists. We can express Eq. (3.3) in per-capita terms, and Eq. (3.5) in per-share terms:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = s'_{WW} \frac{W}{K} + s'_{WC} \frac{P_W}{K} + s'_C \frac{P_C}{K} - \frac{\dot{M}_C}{PK} - \frac{\dot{M}_W}{PK} - g \tag{3.15}$$

$$\frac{\dot{k}_C}{k_C} = \frac{\dot{K}_C}{K_C} - \frac{\dot{K}}{K} = s'_C \frac{P_C}{K_C} - \frac{\dot{M}_C}{PK_C} - \frac{\dot{k}}{k} - g \tag{3.16}$$

Taking into consideration a well-behaved neoclassical production function,  $y = Y/L = f(k)$ , ( $f'(k) > 0, f''(k) < 0$ ), the rental price of capital ( $r$ ) corresponds to the marginal productivity of capital,  $r = f'(k)$ . By the Euler theorem, the wage rate is given by:  $w = f(k) - kf'(k)$ . So the profits are  $P_W = rK_W, P_C = rK_C$ . Using Eqs. (3.8), (3.9), (3.10), (3.11), (3.12), (3.13) and (3.14), we can rewrite Eqs. (3.15) and (3.16) as:

$$\begin{aligned} \frac{\dot{k}}{k} = & \left( \frac{f(k)}{k} - f'(k) \right) s'_{WW} + s'_{WC} f'(k) (1 - k_C) + s'_C f'(k) k_C - g \\ & - \left( \frac{\dot{z}}{z} + \mu \right) k_C \delta(k, \pi) - \left( \mu - \frac{\dot{z}}{1 - z} \right) (1 - k_C) \beta(k, \pi) \end{aligned} \tag{3.17}$$

$$\frac{\dot{k}_C}{k_C} = s'_C f'(k) - \left( \frac{\dot{z}}{z} + \mu \right) \delta(k, \pi) - \frac{\dot{k}}{k} - g \tag{3.18}$$

The dynamic system in Eqs. (3.17) and (3.18) resumes the relevant information on the two-class monetary growth model with public sector.

The terms  $\dot{z}/z$  and  $\dot{z}/(1 - z)$  are defined as (see Appendix 2 for their derivation):

$$\frac{\dot{z}}{z} = g + \pi - \mu + \frac{\dot{k}_C}{k_C} + \left( 1 + k \frac{\delta_k}{\delta} \right) \frac{\dot{k}}{k} + \frac{\delta_\pi}{\delta} \dot{\pi} \tag{3.19}$$

$$\frac{\dot{z}}{1 - z} = \mu - \pi - g + \frac{\dot{k}_C}{1 - k_C} - \left( 1 + k \frac{\beta_k}{\beta} \right) \frac{\dot{k}}{k} - \frac{\beta_\pi}{\beta} \dot{\pi} \tag{3.20}$$

Introducing Eqs. (3.19) and (3.20) into Eq. (3.17), and Eq. (3.19) into Eq. (3.18), and assuming that inflation is a positive constant:  $\pi = \bar{\pi} \geq 0$ , which makes  $\dot{\pi} = 0$ . In the steady-state  $\dot{k} = \dot{k}_C = \dot{z} = 0$ , the long-run solutions (denoted by asterisk) for  $k, k_C$ , are given by the following equations:

$$\mu = g + \pi \quad (3.21)$$

$$s'_C f'(k^*) - (g + \pi)\delta(k^*, \pi) - g = 0 \quad (3.22)$$

$$s'_{WW} \left( \frac{f(k^*)}{k^*} - f'(k^*) \right) + s'_{WC} f'(k^*) (1 - k^*_C) + s'_C f'(k^*) k^*_C - g - (g + \pi) k^*_C \delta(k^*, \pi) - (g + \pi) (1 - k^*_C) \beta(k^*, \pi) = 0 \quad (3.23)$$

note that Eq. (3.21) must hold as a long run macroeconomic equilibrium in which the rate of monetary growth is equal to the rate of population growth plus the rate of inflation.

From Eq. (3.22) we can derive the Cambridge equation:

$$r^* = f'(k^*) = g(1 + \delta(k^*, \pi))/s'_C + \pi\delta(k^*, \pi)/s'_C \quad (3.24)$$

It is a remarkable fact, given by Eq. (3.24), that the Pasinetti paradox continues to be one of the solutions, despite fiscal and monetary extensions of the model. It means that workers' savings, investments, taxation and demand for money do not affect the rate of profit of the economy and the accumulation of capital. Notice, however, that inflation, together with the capitalists money demand, and corrected marginal propensity to save of the capitalists, have an important role in the determination of the rate of profit and in the accumulation of capital.

In order to do a comparative statics analysis of the Cambridge equation, we have to bear in mind that it should be a stable equilibrium. Following the methodology proposed by Faria and Teixeira (1999) one can provide the necessary and sufficient conditions for the stability of this equilibrium.

There are two interesting cases to be analyzed. The first one is the case where the price level is kept constant, which implies  $\pi = 0$ . Then, Eq. (3.24) becomes:

$$r^* = f'(k^*) = g(1 + \delta(k^*))/s'_C \quad (3.25)$$

By Eq. (3.25) only fiscal policy matters for the long-run properties of the model. Fiscal policy affects the rate of profit and the accumulation of capital in the economy through the corrected marginal propensity to save of the capitalists ( $s'_C$ ). If  $s'_C$  increases, the accumulation of capital increases and the rate of profit decreases. The former result can be seen by the comparative static analysis of Eq. (3.25):

$$\frac{dk}{ds'_C} = \frac{-g(1 + \delta(k))}{s'_C f''(k) - g\delta_k(k)} > 0 \quad (3.26)$$

According to Eq. (2.14'), the term  $s'_C$  depends on the proportional tax on profits ( $t_P$ ). We can see below how this term affects negatively the corrected marginal propensity to save of the capitalists, and then, how they impact on the accumulation of capital and the rate of profit.

The fiscal policy impact on  $s'_C$  is given by the derivation of Eq. (2.14'):

$$\frac{ds'_C}{dt_P} = -s_C < 0 \quad (3.27)$$

by Eqs. (3.27) and (3.26), an increase in the proportional tax on profits actually decreases the accumulation of capital and increases the rate of profit of the economy, because:

$$\frac{dr}{dt_P} = \frac{dr}{dk} \frac{dk}{ds'_C} \frac{ds'_C}{dt_P} > 0$$

The second case to be investigated is when the inflation rate is positive. The Cambridge equation is given by Eq. (3.24). In this case, inflation can have a positive, negative or no effect on the profit rate and also on the accumulation of capital as seen below:

$$\frac{dr}{d\pi} = \frac{(g + \pi)\delta_\pi + \delta(k, \pi)}{s'_C} \leq 0 \Leftrightarrow \mu = g + \pi \leq \frac{-\delta(k, \pi)}{\delta_\pi} \tag{3.28}$$

$$\frac{dk}{d\pi} = \frac{(g + \pi)\delta_\pi + \delta(k, \pi)}{f''(k)s'_C - g\delta_k(k, \pi)} \leq 0 \Leftrightarrow \mu = g + \pi \leq \frac{-\delta(k, \pi)}{\delta_\pi} \tag{3.29}$$

Eqs. (3.28) and (3.29) exhibit ambiguous effects of inflation on growth. They show that inflationary monetary policies do not necessarily rise the accumulation of capital. The power of the inflationary monetary policy depends on capitalists' demand for money and the sensitivity of their demand for money in relation to the rate of inflation, as well. This result is interesting because it leads to the possibility of a non-linear relationship between inflation and growth<sup>4</sup>. It means that, for particular inflation rates, inflation and growth are positively related, and for others it is negatively related. However, we are not able to determine theoretically the intervals above. The impact of fiscal policy in the case of positive inflation is the same as in the case of no inflation.

#### 4. Concluding remarks

It has been shown in this paper that fiscal and monetary policies do not affect the nature of the Cambridge equation. Our model modifies and extends Ramanathan (1976) approach to the case in which fiscal policy is a significant feature of the economy along the lines of Pasinetti (1989). The model developed in terms of a system of differential equations presents the Cambridge equation as one of the steady-state solutions.

The analysis of the impact of fiscal policies on the long-run equilibrium shows that a rise in the proportional tax rate on profits causes an decrease in the accumulation of capital and an increase in the rate of profit. On the other hand, an inflationary monetary policy is shown to have ambiguous effects on the accumulation of capital and on the rate of profit of the economy. The power of an

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<sup>4</sup> Thirlwall (1974), and more recently, Sarel (1996), have discussed empirical findings which associate inflation and growth in a non-linear way. They have shown that inflation and growth are positively related for small rates of inflation and negatively related for high inflation rates.

inflationary monetary policy depends on capitalists' demand for money and the sensitivity of the demand for money of capitalists to inflation. This result means that there is a possible non-linear relationship between inflation and economic growth.

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### Appendix 1. Basic notation

$C$	consumption
$g$	harrodian 'natural rate' of growth, and population rate of growth
$G$	government expenditure
$I$	investment
$k$	capital per head
$K$	aggregate capital stock
$k_C$	capitalists' share of the capital stock
$K_C$	capital owned by capitalists
$k_W$	workers' share of the capital stock
$K_W$	capital owned by workers
$L$	labour force (total)
$m \equiv \frac{M}{P}$	real money balances
$m_C$	capitalists' real money balances
$m_W$	workers' real money balances
$P_C$	profits accruing to the capitalists
$P_W$	profits accruing to the workers
$r$	rate of profit on capital
$R$	real wealth
$S$	total savings
$s_C$	capitalists' (marginal and average) propensity to save
$S_C$	capitalists' net savings of profits taxes
$s_g$	proportion of total taxes that is not spent
$S_g$	government savings
$s_W$	workers' (marginal and average) propensity to save
$S_W$	workers' net savings of both wages and profits taxes
$T$	total taxation
$t_i$	proportional (indirect) tax on total consumption expenditures of all individuals (workers and capitalists) and of the government itself
$t_P$	proportional (direct) tax on profits
$t_W$	proportional (direct) tax on wages
$W$	wages

$Y$	national income
$z$	proportion of total real money balances held by capitalists
$\mu$	instantaneous rate of monetary growth
$\pi$	inflation rate

Dotted variables represent time derivatives

## Appendix 2

Derivation of Eq. (3.20). From Eq. (3.14) we have:

$$M_C = pK_C \delta(k, \pi) \quad (\text{A.1})$$

Differentiating Eq. (A.1) in relation to the time, we obtain:

$$\dot{M}_C = \dot{p}K_C \delta(\cdot) + p\dot{K}_C \delta(\cdot) + pK_C [\delta_k \dot{k} + \delta_\pi \dot{\pi}] \quad (\text{A.2})$$

Dividing Eq. (A.2) by  $M_C$ , and using Eq. (A.1), follows:

$$\frac{\dot{M}_C}{M_C} = \pi + \frac{\dot{k}}{k} + \frac{\dot{k}_C}{k_C} + g + \frac{\delta_k}{\delta} \dot{k} + \frac{\delta_\pi}{\delta} \dot{\pi} \quad (\text{A.3})$$

Noticing that:

$$\frac{\dot{M}_C}{M_C} = \frac{\dot{z}}{z} + \mu \quad (\text{A.4})$$

Substituting Eq. (A.4) into Eq. (A.3) and rearranging, Eq. (3.20) follows.

For the derivation of Eq. (3.21), notice first that from Eq. (3.15) we have:

$$M_W = pK_W \beta(k, \pi) \quad (\text{A.5})$$

Differentiating Eq. (A.5) in relation to time, we obtain:

$$\dot{M}_W = \dot{p}K_W \beta(\cdot) + p\dot{K}_W \beta(\cdot) + pK_W [\beta_k \dot{k} + \beta_\pi \dot{\pi}] \quad (\text{A.6})$$

Dividing Eq. (A.6) by  $M_W$ , using Eq. (A.5), and noticing that:

$$\frac{\dot{M}_W}{M_W} = \mu - \frac{\dot{z}}{1-z} \quad (\text{A.7})$$

Eq. (3.21) follows.

## References

- Darity, W., 1981. A simple analytics of neo-ricardian growth and distribution. *Am. Econ. Rev.* 71, 978–993.
- Faria, J.R., Teixeira, J.R., 1999. Growth and stability in a model with Pasinettian saving behaviour and neoclassical technology. *Manchester Sch.* 67, 111–121.
- O'Connell, J., 1995. The two/one class model of economic growth. *Oxford Econ. Pap.* 47, 363–368.

- Pasinetti, L., 1989. Ricardian debt/taxation equivalence in the Kaldor theory of profits and income distribution. *Cambridge J. Econ.* 13, 25–36.
- Ramanathan, R., 1976. The Pasinetti paradox in a two-class monetary growth model. *J. Monetary Econ.* 2, 389–397.
- Samuelson, P., Modigliani, F., 1966. The Pasinetti paradox in neoclassical and more general models. *Rev. Econ. Stud.* 33, 269–301.
- Sarel, M., 1996. Nonlinear effects of inflation on economic growth. *IMF Staff Pap.* 43, 199–215.
- Sidrauski, M., 1967. Inflation and economic growth. *J. Polit. Econ.* 75, 798–810.
- Thirlwall, A.P., 1974. *Inflation, Saving and Growth in Developing Economies*. Macmillan, London.
- Tobin, J., 1955. A dynamic aggregative model. *J. Polit. Econ.* 63, 103–115.