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Terror Cycles

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Abstract

This paper makes two contributions to the literature on terrorism: 1) It presents a model that explains the cyclical characteristics of terrorist attacks, and 2) It improves on the existing theoretical cyclical models since it takes explicitly into account terrorists' motivations and decision-making. The model assumes that the government acts as leader and terrorists as followers. Terrorists maximize terrorist acts over time constrained by their budget constraint, which yields a time path for terrorist activities. Then the government maximizes national security by taking the time path of terrorist activities and investments in enforcement as constraints. The model shows that permanent cyclical paths for enforcement, terrorist activities, terrorist capital and national security may arise as an optimal solution.

1. Introduction

The 9/11 attacks have attracted the media attention to terrorism activities. However, academic literature has been studying terrorism for decades. Despite the growth of religious terrorism since 1991 as noticed by Enders and Sandler (2000), terrorism acts have been targeting and having major effects on tourism destinations since before the end of the Cold War[1] [Pizam and Smith, 2000]. Empirical studies [e.g., Enders et al, 1992a] calculate the impact of terrorism on tourism and show that terrorism lead to large revenue losses. The substantial economic costs of terror are not solely related to the tourism industry[2]. Enders and Sandler (1996) show that terrorism has a significant and persistent negative impact on net foreign direct investment and it can reduce investment and growth for smaller countries that face a persistent threat of terrorism.

An important empirical regularity found by Enders et al. (1992b) about terrorist time series involving all incidents, hostage incidents, bombings, assassinations, and threats and hoaxes during the period 1970-1989, is that cycles characterize all of the series. These authors argue that there is a clear need to develop a theoretical framework capable of explaining these cycles. Feichtinger et al. (2001) provide a model where a persistent cycle is derived for terrorist activities and tourism industry. The model shows that enforcement to fight terrorism, investment in the tourism industry and the number of tourists and terrorists display a cyclical path.

The workings of the Feichtinger et al. (2001) model are as follows. The starting point is a country with a small tourism industry. Then the country starts to invest in tourism, which attracts tourists. By increasing the number of tourists it attracts terrorists, which increases enforcement. Eventually, terrorism activities distract tourists from visiting the country, lowering the investments in the tourism industry. The fall in the number of tourists and investments lead to a decrease in terror acts, which decreases enforcement, restoring the initial situation and the cycle repeats itself.

In Feichtinger et al (2001) model the government is the decision maker and its objective is to maximize the income generated by the tourism industry. The number of terrorists is regarded as a state variable, and there is no analysis on how terrorist decisions are made. Many models employ a game theoretical framework to address the relationship between government and terrorists, because it is assumed that both agents are rational players and their decisions may affect the decisions of the opponent [e.g., Sandler et al., 1983; Lee, 1998]. That is, it is necessary to make a microeconomic analysis of terrorist organizations[3]. This paper fills this gap by presenting a cyclical model of terror where the decision making of terrorists is explicitly taken into account.

In this paper the government is assumed to be the leader and the terrorists are the follower. In this sense, terrorists aim at maximizing their attacks subject to their dynamic budget constraint. Terrorism activities are costly, investing in terrorist capital such as weapons and bombs involve not only financial resources, but also strategic resources to

[1] O'Brien (1996) shows that, before the end of the Cold War, the Soviet Union used international terrorism as a foreign policy tool.
[2] Glaeser and Shapiro (2002) analyze the impact of large-scale violence in urban form. However, they do not find significant effect of terrorism on American cities.
[3] Gates (2002) theorizes about rebel organizations using a principal-agent type model.

deal with police repression. Police repression and any other type of law enforcement is taken as given by terrorists. The solution of the terrorist's problem yields a dynamic equation for terrorist activities. It follows that the government, acting as leader, takes the time path of terrorist acts as a state equation and solves a constrained maximization problem where it aims at maximize national security, subject to enforcement investments and terrorist activities. The solution of this problem yields a limit cycle between enforcement and terrorist activities. As a consequence, there is a persistent cycle between enforcement and terror, which leads to national security and terrorists investment to display cyclical paths as well.

Therefore, this paper makes two contributions to the literature. First, it provides a theoretical model to explain the observable cyclical pattern of terrorist attacks as registered by Enders et al. (1992b). Second, it improves on the existing theoretical model of Feichtinger et al. (2001), by allowing the analysis of terrorist decisions.

The contents of the paper are as follows. In section 2, the model is presented. In section 3, the limit cycle is analyzed. Finally, the concluding remarks appear in section 4.

2. The Model

2.1 The Terrorist Problem

The terrorist model below is a standard microeconomic model, where the representative terrorist group solves a maximization problem based on preferences, actions, incentives and budget and strategic restrictions[4]. The terrorist organization is treated as a unitary agent, i.e., the model abstracts from problems of collective choice[5]. Terrorists derive satisfaction from terrorist activities (A). Terrorist activities are a denomination general enough to include assassination, kidnapping, bombing, hijacking and any other form of terrorist activity. The satisfaction (U) that terrorists derive from terror can have different sources; they can be based on political, religious or racial motivations[6]. The preferences of the representative terrorist organization for terrorist acts are represented by a strictly concave monotonically increasing instantaneous utility function $U(A)$.

Terrorist activities are costly. Terrorists need to build a network that allows them to pay for the resources they need to perform their acts. The amount of resources employed to perform terrorist acts is called terrorist capital (k). Terrorists need resources to pay for the investment in terrorist capital ($\dot{k}$), these resources are denoted as (y), which is kept as an exogenous variable in order to capture the effect of financial networks. The investment in terrorist capital is also affected by law enforcement. Enforcement is represented by the enforcement stock (E). All this is captured by the investment function $\dot{k} = G(k, y, A, E)$. Of course, enforcement has a negative impact on the investment of terrorist capital: $G_E < 0$. In addition, it seems reasonable to assume that $G(k, y, A, E)$ has the following characteristics:

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[4] Rathbone and Rowley (2002) provide a length discussion on the nature, goals, incentives, and costs of terrorism.

[5] This is a usual working hypothesis [e.g., Gershenson and Grossman, 2000]. On the role of leadership organizing collective action in civil conflicts see, among others, Tullock (1974) and Grossman (1999).

[6] Holmes (2001) stresses the importance of public support for terrorist activities.

$$G_k > 0, G_{kk} > 0, G_A < 0, G_{AA} = 0, G_y > 0, G_{yy} = 0, G_{ky} > 0,$$
$$G_{kA} = 0, G_{yA} = 0, G_{AE} = 0, G_{kE} < 0 \qquad (1)$$

The first and second inequalities of (1) state that investment in terrorist capital is convex in terrorist capital. This captures the idea that more sophisticate weaponry demands more investment. The third and fourth inequalities mean that there is a trade-off between implementing terrorist attacks (*A*) and investing in terrorist capital. Inequality fifth and equality sixth simply say that terrorists need financial resources to invest. Inequality seventh says that the positive effect on investment of an additional unit of capital is increasing with the availability of financial resources. The following three equalities state that there is no marginal effect of *A* on the positive effect of capital and financial resources, and on the negative effect of law enforcement. The last mixed derivative say that the positive effect on investment of an additional unit of capital is decreasing with law enforcement.

The net gain of terrorism at any point of time corresponds to the difference between the benefits and costs of terrorist activities, and is given by the following function: $V(A, \dot{k}, k, t) = U(A) + \lambda[\dot{k} - G(k, y, A, E)]$, where the multiplier λ is the shadow price of terrorist capital. Applying the discount factor $e^{-\theta t}$ [where θ is the terrorist's rate of time preference] to this expression and summing over time, we can express the present-value net gain of terrorism as[7]:

$$Max \int_0^{\infty} \{U(A) + \lambda[\dot{k} - G(k, y, A, E)]\} e^{-\theta t}\, dt \qquad (2)$$

The solution of this problem yields an optimal time path for terrorist activities as a function of terrorist capital, financial and other exogenous resources, previous terrorist activities, terrorists time preference and law enforcement:

$$\dot{A} = \Omega(k, y, A, E, \theta) \qquad (3)$$

2.2 The Government Problem

As discussed in the introduction, the government is considered to behave as a leader. In this sense it takes the reaction function of terrorists, given by equation (3), into consideration. The government goal is to maximize the national security (*S*). In order to achieve this objective the government invests in enforcement ($\dot{E}$). The investment in enforcement can take different forms, such as increasing security personnel, installation of metal detectors in airports, creation of "get-tough" laws on terrorism, etc[8]. The investment in enforcement grows with terrorist threats and with terrorist activities. This effect is amplified by the available structure of enforcement. Of course, the higher the security level achieved, less investment will be made in enforcement. These

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[7] It must be assumed that $k > 0$, otherwise one could make A arbitrarily high, leading to an infinite utility functional.

[8] See an empirical study to assess the effectiveness of specific terrorist-thwarting policies in Cauley and Im (1988) and Enders et al. (1990) .

considerations allow us to write the investment in enforcement $\dot{E} = \Theta(A, E, S)$ as a function with the following characteristics:

$$\Theta_A > 0,\ \Theta_{AA} < 0,\ \Theta_E > 0,\ \Theta_{EE} = 0,\ \Theta_{AE} > 0,\ \Theta_S < 0, \Theta_{SS} = 0, \Theta_{AS} < 0 \qquad (4)$$

Therefore the government intertemporal problem is[9]:

$$\begin{aligned} &\underset{S}{Max} \int_0^\infty V(S) e^{-\rho t}\, dt \\ &s.t.\ \dot{E} = \Theta(A, E, S) \\ &\qquad \dot{A} = \Omega(k, y, A, E, \theta) \end{aligned} \qquad (5)$$

where V is a strictly concave monotonically increasing instantaneous utility function and the parameter ρ is the government rate of time preference, which is assumed to be positive.

The analysis of the government problem, presented in the next section, shows that one of the possible solutions is a limit cycle between enforcement (E) and terrorist attacks (A). This can lead to other endogenous variables in the model, such as national security (S) and terrorist capital (k), to display permanent cyclical paths as well.

3. The Limit Cycle

In order to solve the model presented in the last section we must use specific functions. Assuming that the investment in terrorist capital is given by:

$$\dot{k} = G(k, y, A, E) = \frac{a k^2}{2E} y - bA \qquad (2')$$

and that the instantaneous utility function of terrorists is given by: $U(A) = \ln A$, it is possible to solve the terrorist problem by using the calculus of variations theory.

The terrorist's organization objective is to maximize the present value of the net gain of terrorism by choosing an optimal A path and an optimal k path :

$$\underset{A,k}{Max} \int_0^\infty \{U(A) + \lambda \left[\dot{k} - \left(\frac{ak^2}{2E} y - bA \right) \right] \} e^{-\theta t}\, dt$$

There are two state variables, A and k, in the objective functional. There will be two Euler equations yielding two optimal paths:

$$V_A = 0 \Rightarrow \lambda = -(bA)^{-1} \qquad (7)$$

$$V_k - \frac{d}{dt} V_{\dot{k}} = 0 \Rightarrow \dot{\lambda} = \lambda \left[\theta - \frac{ak}{E} y \right] \qquad (8)$$

By deriving (7) in relation to time and introducing it in (8) yields the time path for terrorist activities:

$$\dot{A} = A \left(\frac{ak}{E} y - \theta \right) \qquad (3')$$

[9] The nonnegativity constraint $E \geq 0$ has to hold, otherwise it would always be possible to achieve an infinitely high utility by choosing a sufficiently high level of security *S*.

The government acts as a leader and takes equation (3') as a constraint for the maximization of national security. By specifying the investment in enforcement [in accordance with (4)] as:

$$\dot{E} = A(\delta E - \beta AS) \tag{9}$$

and assuming that the government instantaneous utility function is: $V(S) = \ln S$, it is possible to solve the government problem.

The Hamiltonian corresponding to the government constrained maximization problem is:

$$H = \ln S + \gamma[A(\delta E - \beta AS)] + \psi[A(akyE^{-1} - \theta)]$$

where the co-state variables γ and ψ indicate the shadow prices of enforcement and terrorist attacks, respectively. The first order conditions are:

$$H_S = 0 \Rightarrow S = (\gamma \beta A^2)^{-1} \tag{10}$$

$$\dot{\gamma} = \gamma(\rho - \delta A) + \psi AakyE^{-2} \tag{11}$$

$$\dot{\psi} = \psi[\rho - (aky E^{-1} - \theta)] - \gamma(\delta E - 2\beta AS) \tag{12}$$

To analyze the stability behavior of this model we first introduce equation (10) into equations (9) and (12) which yields equations (13) and (16) below. The dynamic system of our interest is:

$$\dot{E} = A[\delta E - (\gamma A)^{-1}] \tag{13}$$

$$\dot{A} = A(aky E^{-1} - \theta) \tag{14}$$

$$\dot{\gamma} = \gamma(\rho - \delta A) + \psi AakyE^{-2} \tag{15}$$

$$\dot{\psi} = \psi[\rho - (aky E^{-1} - \theta)] - \gamma[\delta E - 2(\gamma A)^{-1}] \tag{16}$$

The steady state solutions (denoted by an asterisk) are found by making $\dot{A} = \dot{E} = \dot{\gamma} = \dot{\psi} = 0$:

$$E^* = aky\theta^{-1} \tag{17}$$

$$A^* = \rho^2[\delta(\theta + \rho)]^{-1} \tag{18}$$

$$\gamma^* = [\theta(\theta + \rho)][\rho^2 aky]^{-1} \tag{19}$$

$$\psi^* = -\rho^{-3}[\delta(\theta + \rho)] \tag{20}$$

Notice that by equation (20) the equilibrium shadow price of terrorist attacks ψ^* is negative, since terrorism is a menace to national security. As expected, equation (17) shows that the equilibrium enforcement stock (E^*) increases with terrorist spending in terrorist capital [ky]. This means that the government optimal policy is to match any increase in the resources available for terrorist activities by increasing the stock of enforcement. In addition, notice by equation (18) that the number of terrorist activities decreases with terrorists' rate of time preference. That is, higher terrorist impatience leads to less successful terrorist activities.

In order to show that a limit cycle is a solution for the government's problem, let us consider the Jacobian of the system (13)-(16) calculated with the steady state solutions (17)-(20) is the following:

$$J = \begin{vmatrix} \delta A^* & \delta E^* & \gamma^{*-2} & 0 \\ -A^* akyE^{*-2} & 0 & 0 & 0 \\ -2\psi^* A^* akE^{*-3} & -\gamma^* \delta + \psi^* akyE^{*-2} & \rho - \delta A^* & A^* akyE^{*-2} \\ \psi^* akyE^{*-2} - \delta \gamma^* & -2A^{*-2} & -\delta E^* & \rho \end{vmatrix}$$

which equals:

$$J = \theta A^* E^* \{\delta\rho E^* (\rho - \delta A) + \delta^2 A^* aky - \gamma^{*-2} [2\theta (A^* E^*)^{-1} + \rho(\theta\psi^* E^{*-1} - \delta\gamma^*)]\} \tag{21}$$

It is important to stress that J is positive. ogether with the number M, that has the following form:

$$M = \begin{vmatrix} \delta A^* & \gamma^{*-2} \\ -2\psi^* A^* akE^{*-3} & \rho - \delta A^* \end{vmatrix} + \begin{vmatrix} 0 & 0 \\ -2A^{*-2} & \rho \end{vmatrix} + 2\begin{vmatrix} \delta E^* & 0 \\ -\gamma^* \delta + \psi^* akyE^{*-2} & A^* akyE^{*-2} \end{vmatrix}$$

which can be rewritten into:

$$M = \delta A^* (\rho - \delta A^*) + 2E^{*-3} \psi^* A^* aky \gamma^{*-2} + 2\delta A^* akyE^{*-1} \tag{22}$$

The consideration of equations (21) and (22) allows us to derive the main result of this paper, which is announced in the following proposition:

Proposition: If $\theta > \rho$, and$1 > \delta > 0.5$, there is a limit cycle between government enforcement (*E*) and terrorist activities (*A*).

Proof: $\theta > \rho$, and $1 > \delta > 0.5$ imply:

$\theta \delta > \rho(1-\delta) \Rightarrow (\theta + \rho)\delta > \rho \Rightarrow$

$1 > \rho[(\theta + \rho)\delta]^{-1} \Rightarrow \rho > \rho^2 [(\theta + \rho)\delta]^{-1} \Rightarrow \rho > A^* \Rightarrow \rho > \delta A^*$

by equation (22) this makes $M > 0$. As J is always positive, the value of the bifurcation parameter (ρ) given by the condition below:

$$J = \left(\frac{M}{2}\right)^2 + \rho^2 \left(\frac{M}{2}\right)$$

is positive. These three conditions: $J>0$, $M>0$ $\rho > 0$, are sufficient, according to Feichtinger et al (1994), to generate a limit cycle, through the Hopf bifurcation theorem, between terrorist activities and enforcement.

Notice that one of the conditions for the existence of the limit cycle is that the terrorist rate of time preference is greater than the government rate of time preference $\theta > \rho$, that is, terrorists are more impatient than the government. The other condition $1 > \Theta_{AE} = \delta > 0.5$, states that the positive effect on investment in enforcement of an additional unit of enforcement capital is not only increasing with terrorist activities, but must be greater than 0.5, and less than one. This means that the government reacts to terrorist attacks by increasing the investment in enforcement.

Given that terrorist activities are cyclical it leads national security to display a cyclical pattern as well through equation (10). In the same vein, given that E and A are cyclical they cause, through equation (2’), the stock of terrorist capital to display a permanent cyclical trajectory as well.

The description of the cycle is as follows. When the enforcement capital is low, law enforcement is not effective which creates incentives for terrorists to invest in terrorist capital and terrorist activities increase. The raise in terror decreases national security and leads the government to invest in enforcement. By improving enforcement the government forces a reduction in terrorist investments leading to a fall in terrorist activities, increasing national security. When national security is high, the government has an incentive to decrease investment in enforcement, lowering the enforcement capital that triggers the cycle to repeat itself.

4. Concluding Remarks

This paper fills two gaps in the literature on terrorism. It presents a theoretical model that explains the cyclical characteristics of terrorist attacks, which is in line with the available empirical evidence. In addition, it improves on the existing theoretical cyclical models since it takes explicitly into account terrorists’ motivations and decision-making.

The model assumes that the government acts as leader and terrorists as followers. Terrorists maximize terrorist acts over time constrained by their budget constraint. The budget constraint is affected by terrorist networks, choice in resource allocation and strategies in dealing with enforcement. The solution of the terrorist problem yields a time path for terrorist activities. The government takes the time path of terrorist activities into account when maximize national security over time. It also considers the investments in enforcement as a constraint. The investments in enforcement describe the strategies of the government in dealing with the terrorist threat. The solution of the government problem yields a limit cycle between enforcement and terrorist activities. The permanent cyclical paths in enforcement and terror cause national security and terrorist stocks to display cyclical trajectories as well.

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