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RAMSEY IN DUAL-POPULATION LANDS: INTERNAL CONFLICT AND UTILITY-MAXIMIZING CONSUMPTION

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Ramsey's model is extended to three possible scenarios of conflicts in dual-population lands: partition, federation and civil war. The federally utility-maximizing consumption-growth rate in a strictly political federation might be lower than that under partition for the wealthier and more slowly multiplying group. This group may benefit from joining a federation that facilitates technological transfer and from obeying the federal no-arbitrage rule as long as its own technology is inferior to the hybrid. The utility-maximizing consumption growth rate for a group engaged in a civil war is larger than those under partition and a strictly political federation if its rival's warfare is mainly aimed at inflicting casualties and is likely to be smaller when its rival's warfare is mainly sabotage.

Keywords: Internal conflict; Partition; Federation; Civil war; Utility-maximizing consumption

JEL CODES: D90, D72, D74, J19, P16

INTRODUCTION

Ramsey's (1928) seminal paper on optimal consumption and capital accumulation was written during the last phase of the colonial era. The present paper speculates how Ramsey's model would have been modified had Ramsey anticipated the internal conflicts that are currently affecting the consumption, capital accumulation and stability of many post-colonial countries and also their former rulers.

The populations of many former colonial lands are divided by factors such as origin, culture, religion and race into two major groups. Notable examples are Hindus and Muslims in the Indian sub-continent, Muslims and Christians in Sudan, Nigeria, Ethiopia and Lebanon, Greeks and Turks in Cyprus, Jews and Palestinians in Israel–Palestine, Muslims and Orthodox Serbs in Bosnia Herzegovina, Hutus and Tutsis in Rwanda, Pushtuns and Tadzhiks in Afghanistan, Sinhalese and Tamils in Sri Lanka, Indians and Melanesians in Fiji, and Blacks and Whites in southern Africa. Conflicts between the

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groups inhabiting these lands seem to be imminent and constitute a major aspect of their coexistence.

Indeed, the literature on internal conflicts is mainly focused on civil wars. Collier and Hoeffler (1998, 2000) argue that civil wars break out when rebels' perceptions of benefits outweigh the costs of rebellion and are motivated either by greed for private gains or by grievance stemming from the degree of autocracy of the regime and from ethnic and religious differences. Elbadawi and Sambanis (2002) find that civil violence is negatively associated with democracy and that civil-war prevalence is positively associated with ethnic fragmentation. Reynal-Querol (2002) concludes that religious differences constitute a social cleavage that is more important than linguistic differences in the development of civil wars and that democracy significantly reduces the incidence of civil wars between ethnic groups.

There may be a less painful resolution to internal conflicts in dual-population lands. When the rival groups are similar in size and posses a similar technology and material wealth the costs of civil war for each group are high. In this case, a partition of the land (if practically possible) between the rival groups may settle their conflict with the least cost. Moreover, when the cultural differences between the groups are small, when the degree of cohesion of each group is low and when the level of human and material capital disparity between the groups is low, the probability of a formation of a federation, or even integration, is high. There may also be external factors affecting the interaction between the two rival groups and, subsequently, their political equilibrium. On the one hand, external factors such as close relationship with the motherlands might polarize a local population by strengthening ethnic identity (e.g. Greek and Turkish Cypriots). On the other hand, external factors such as a threat of invading peoples may moderate groups' perception of the magnitude and importance of their socioeconomic differences and increase their degrees of mutual tolerance and solidarity. Facing a common enemy, rival groups may form a federation and encourage integration. Left alone, they might prefer partition, or fight one another (e.g. Athenians and Spartans in ancient Greece).

The objective of this paper is to extend Ramsey's (1928) analysis of utility-maximizing consumption and capital accumulation to the three possible types of resolution of conflicts in dual-population lands: partition, federation and civil war. The analysis focuses on the role of population-growth and wealth disparities and compares the utility-maximizing consumption growth rates across the three political scenarios. This comparison sheds light on the conditions under which one of the political scenarios may be preferred to the others by at least one of the rival groups.

The presentation of the political scenarios and their implications for consumption starts with partition. The description of this case in the next section serves as a benchmark for the analytically more complicated scenarios of federation and civil war. The third section analyzes the case of federation and its implication for consumption when the federation is strictly political and, alternatively, when the federation provides economic benefits through technological exchange, and also when the federation facilitates a flow of capital and labor between the two groups. The fourth section analyzes civil-war scenarios and their implications for consumption with a distinction between casualty-intensive (bloody) warfare and capital-destruction-intensive (sabotage) warfare. The final section concludes.

PARTITION

Consistent with Ramsey (1928), our presentation of this case is based on the following assumptions.

Assumption 1 (demographics). The land is inhabited by two groups (i = 1,2). The groups are highly cohesive (i.e. the number of intermarriages is negligible) and the size of each group is given by an exponential growth function:

$$P_i(t) = P_i^0 e^{n_i t} \tag{1}$$

where P_i^0 and n_i are the initial size and the population growth rate of the ith group, respectively.

Assumption 2 (intolerance and deterrence). There are considerable cultural and spiritual differences between the groups, and at least one of the groups prefers a split to coexistence under a single political-economic system. None of the groups is relatively very poor in human and material capital and can be easily subjected by the other.

Assumption 3 (utility). The groups are homogeneous with regard to preferences. The instantaneous utility from consumption (c) of a member of group i is given by $u_i(c_i(t))$, with $u_i' > 0$ and $u_i'' < 0$. For tractability, the explicit form:

$$u_i = c_i^{\beta_i}, 0 < \beta_i < 1 \tag{2}$$

is considered. The member's lifetime utility is additively separable in the instantaneous utilities and displays a non-negative invariant time-preference rate, ρ_i.

Assumption 4 (production and income). The groups are homogeneous with regard to production. Under separation, the aggregate output of each group is given by a Cobb–Douglas production function homogeneous of degree 1 in labor and capital and satisfying Inada's conditions. Consequently, the income of a member of each group is a concave function of the capital–labor ratio in the respective group (i.e. $f_i(k_i(t))$ with $f_i' > 0$ and $f_i'' < 0$).

Assumption 5 (capital accumulation). Capital is linearly depreciated, which, in conjunction with assumptions 1 and 4, implies that, under separation, the instantaneous change in the capital of a member of each group i = 1,2 is given by:

$$\dot{k}_i(t) = f_i(k_i(t)) - c_i(t) - (\delta_i + n_i)k_i(t) \tag{3}$$

where δ_i is the capital depreciation rate in group i.

In view of Assumptions 1 and 2, the costs of civil war are high, but also the costs of tolerating coexistence in a federation are high. Hence, the political outcome is a division of the land into two sovereign states or autonomous parts.[1] Under partition, each group's lifetime-utility maximizing consumption change follows the Ramsey no-arbitrage rule:

$$\tilde{c}_i^{part}(t) \equiv \frac{\dot{c}_i(t)}{c_i(t)} = \frac{f_i'(k_i(t)) - (\rho_i + \delta_i + n_i)}{1 - \beta_i} \tag{4}$$

which states that the instantaneous rate of change in the utility-maximizing consumption for each group is equal to the difference between the marginal product and user cost of capital,

[1] Notable examples are the partitions of the Indian sub-continent, Israel–Palestine and Cyprus.

deflated by the degree of concavity (one minus the elasticity) of the instantaneous utility function of the members.

FEDERATION

Three types of federations are considered. The first is a strictly political federation. The second allows a limited economic cooperation – technological transfer between the two groups comprising the federation. The third also allows flow of capital and labor and intermarriages, which may transform the dual-population society into an integrated one.

Strictly Political Federation (SPF)

Assumption 6 (strictly political federation). The two groups form a federation for political reasons *per se*. Each group retains its own identity and there is no (significant) mobility of labor, capital and technology between the groups.

Assumption 7 (*continuation and dissolution*). There is no upper-bound on the federation life expectancy. However, the federation might be dissolved at every instance t with some probability, $\phi(t)$, whose cumulative distribution function is $F(t)$. The probability of continuation (survival) of the federation beyond t (i.e. $\Phi(t) = 1 - F(t)$) diminishes with the wealth-disparity between the two groups as displayed by:

$$\Phi(t) = e^{-\mu[k_1(t) - k_2(t)]^2} \tag{5}$$

where μ is a positive scalar indicating the sensitivity of the federation's existence to wealth-disparity between the groups. The underlying rationale is that wealth-differential intensifies the poorer group's feelings of relative deprivation, and thereby also intensifies political instability. The larger the groups' wealth-differential, the greater the discontent of the poorer one with the federal system.

Assumption 8 (federal social welfare). The instantaneous federal social welfare (FSW) level is given by the sum of the instantaneous utilities of the individuals affiliated to the federation. Recalling from Assumptions 4 and 5 that the members of each group are identical:

$$FSW(t) = P_1(t)u_1(c_1(t)) + P_2(t)u_2(c_2(t)) \tag{6}$$

Assumption 9 (imaginary crossbreed). The representative agent of the federation is, in the absence of intermarriage, an imaginary crossbreed of the two groups. His instantaneous utility, $u(t)$, reflects an equal share in the federation's instantaneous social welfare level and hence is found by dividing the instantaneous federal social welfare by the federation's population:

$$u(t) = \frac{FSW(t)}{P_1(t) + P_2(t)} \tag{7}$$

By substituting equation (6) into equation (7), the instantaneous utility of the imaginary crossbreed is equal to the weighted average of the groups' representatives' instantaneous utilities:

$$u(t)=\left(\frac{P_1(t)}{P_1(t)+P_2(t)}\right)u_1(c_1(t))+\left(\frac{P_2(t)}{P_1(t)+P_2(t)}\right)u_2(c_2(t)) \tag{8}$$

Assumption 10 (expected lifetime utility). The imaginary crossbreed is aware of the fragility of the federation and the possibility of its dissolution. He has a fixed, non-negative, rate of time preference ($\rho \geq 0$). He chooses the consumption trajectories of his composite personality so as to maximize his expected-lifetime utility:

$$J=\int_0^\infty \phi(t)\int_0^t \mathrm{e}^{-\rho\tau}u(\tau)\,\mathrm{d}\tau\,\mathrm{d}t \tag{9}$$

To solve the federal representative imaginary crossbreed's problem, note that, as explained in Appendix A, $J=\int_0^\infty \phi(t)\int_0^t \mathrm{e}^{-\rho\tau}u(\tau)\mathrm{d}\tau\mathrm{d}t=\int_0^\infty \mathrm{e}^{-\rho t}u(t)\Phi(t)\mathrm{d}t$, recall Assumption 5 and express the consumption of each group as:

$$c_i(t)=f_i(k_i(t))-(\delta_i+n_i)k_i(t)-\dot{k}_i(t) \tag{10}$$

By substituting the right-hand side of equation (10) into the imaginary crossbreed's instantaneous utility function and applying Euler's equation, the consumption-growth rate of each group i which maximizes the imaginary crossbreed's expected lifetime utility is given, as explained in a greater detail in Appendix B, by the following federal no-arbitrage rule:

$$\tilde{c}_i^{SPF}(t)\equiv\frac{\dot{c}_i(t)}{c_i(t)}=\frac{f_i'(k_i(t))-(\rho+\delta_i+n_i)}{1-\beta_i}-\frac{(n_j-n_i)p_j(t)}{1-\beta_i}-\frac{\frac{2\mu}{\beta_i}[k_i(t)-k_j(t)]c_i(t)}{1-\beta_i} \tag{11}$$

where j denotes the counterpart group whose population share in the federation is:

$$p_j(t)=P_j(t)/[P_1(t)+P_2(t)] \tag{12}$$

For each of the groups, the first term on the right-hand side of this federal no-arbitrage rule is identical to the Ramsey no-arbitrage rule in partition, but with the groups' average rate of time preference replacing that of the individual group.

The second term on the right-hand side of the federal no-arbitrage rule indicates that, if the jth group's rate of population growth is larger (smaller) than that of the ith group, the effect of the jth group's share in the federation's population on the federally utility-maximizing rate of change of consumption of the ith group is negative (positive) and hence provides an incentive for the ith group to withdraw from (remain in) the federation. When the populations of both groups grow at the same rate, the effect of the jth population share on the federally utility-maximizing ith group's consumption-growth rate is nil.

The third term on the right-hand side of the federal no-arbitrage rule reveals that the wealth-disparity between the members of group i and the members of group j adversely affects the federation's stability and prospect of survival and hence moderates the federally utility-maximizing consumption growth rate of the ith group members.

From equations (11) and (4),

$$\{\rho_i - \rho \overset{>}{\underset{<}{=}} (n_j - n_i)p_j(t) + \frac{2\mu}{\beta_i}(k_i(t) - k_j(t))c_i(t)\} \Rightarrow \{\tilde{c}_i^{SPF}(t) \overset{>}{\underset{<}{=}} \tilde{c}_i^{PAR}(t)\} \tag{13}$$

This implies that group i may be willing to follow the federal dictate (no-arbitrage rule) as long as:

$$\rho_i - \rho > (n_j - n_i)p_j(t) + \frac{2\mu}{\beta_i}[k_i(t) - k_j(t)]c_i(t) \tag{14}$$

However, if, for example, group i is endowed at time t with a larger initial capital–labor ratio and a lower population growth rate than its counterpart and has a rate of time preference that is not considerably larger than that of the imaginary crossbreed, the federally utility-maximizing rate of change in the consumption of its members in a strictly political federation is lower than that under partition and sole sovereignty. Under these circumstances, group i might prefer not to follow the federal dictate and might secede from the federation at that moment.

Federation with Technological Transfer (FWTT)

Let us now relax a part of Assumption 6 and allow exchange of technology between the two groups comprising the federation. Movement of labor and capital remains prohibited.

Assumption 11 (cost-free and perfect technological transfer). Capital does not become obsolete by technological transfer and adjustment costs are negligible. The two groups perfectly and immediately exchange technological knowledge.

This assumption and (the earlier postulated) rational behavior imply that the two groups use a common hybrid technology, f, since $f(k_i(t)) \geq f_i(k_i(t))$ for each group $i = 1,2$ at every instance.

In this case, the consumption growth rate of each group i that maximizes the imaginary crossbreed's expected lifetime utility is given by the following federal no-arbitrage rule:

$$\tilde{c}_i^{FWTT}(t) \equiv \frac{\dot{c}_i(t)}{c_i(t)} = \frac{f'(k_i(t)) - (\rho + \delta_i + n_i)}{1-\beta_i} - \frac{(n_j - n_i)p_j(t)}{1-\beta_i} - \frac{\frac{2\mu}{\beta_i}[k_i(t) - k_j(t)]c_i(t)}{1-\beta_i} \tag{15}$$

In contrast to the strictly political federation case, the members of group i may economically benefit from staying in the federation and following the federal no-arbitrage rule, even when they are wealthier and multiply in a lower rate than their counterparts, as long as they significantly gain from the technological transfer. More formally, if

$$(\rho_i - \rho) > (n_j - n_i)\rho_j(t) + \frac{2\mu}{\beta_i}[k_i(t) - k_j(t)]c_i(t) \tag{16}$$

and

$$f'(k_i(t)) - f_i'(k_i(t)) + (\rho_i - \rho) > (n_j - n_i)p_j(t) + \frac{2\mu}{\beta_i}[k_i(t) - k_j(t)]c_i(t) \tag{17}$$

then

$$\tilde{c}_i^{FWTT}(t) > \tilde{c}_i^{PAR}(t) > \tilde{c}_i^{SPF}(t) \tag{18}$$

Federation with a Broad Economic Mobility and Social Integration

The case of a broad economic mobility and social integration requires a further relaxation of Assumption 6 to allow capital and labor flows and a relaxation of Assumption 1 to allow intermarriages. Significant, continuous flow of capital and labor and intermarriages can be interpreted as a process of *integration* of the two groups. As argued in the introduction, integration may take place when the cultural and the human and material-wealth differences between the groups are small and when each group is not highly cohesive (in which case, $\mu \to 0$). When integration starts, the crossbreed is no longer imaginary. When integration is completed everybody is a crossbreed. The utility-maximizing consumption growth rate of the society of crossbreeds is given by the Ramsey no-arbitrage rule — the first term on the right-hand side of equation (14) but with time-preference rate, population-growth rate, depreciation rate and technology characterizing the society of crossbreeds.

CIVIL-WAR

In our expansion of Ramsey's model to the case of civil war (CW), each group i makes offenses against the other and takes into account that its own effort in carrying hostile activities (h_i) increases (due to hatred and/or revenge-seeking) its satisfaction, but adversely affects its capital accumulation due to divergence of resources from production to warfare activity. In the same vein, the effort of the antagonist (h_j) inflicts casualties on group i (i.e. reduces the population growth of group i), and damages its current capital stock (i.e. accelerates the depreciation rate of the capital stock of group i). These aspects of civil-war are formally presented by the following assumptions.

Assumption 12 (war-time utility). In addition to, and separately from, satisfaction from consumption, each group generates instantaneous utility from carrying hostile activities against its adversary. That is:

$$u_i(t) = u_i(c_i(t), h_i(t)) \tag{19}$$

where $\frac{\partial u_i}{\partial c_i} > 0, \frac{\partial^2 u_i}{\partial c_i^2} < 0, \frac{\partial u_i}{\partial h_i} > 0, \frac{\partial^2 u_i}{\partial h_i^2} < 0, \frac{\partial^2 u_i}{\partial h_i \partial c_i} = 0 = \frac{\partial^2 u_i}{\partial c_i \partial h_i}$.

Assumption 13 (war-time capital accumulation). The war effort reduces the ith group's capital-investment possibilities. In addition, the hostile actions carried by its rival group adversely affect the ith group's population growth and capital stock. More specifically:

$$\dot{k}_i(t) = f_i(k_i(t)) - c_i(t) - h_i(t) - [\delta_i(h_j(t)) + n_i(h_j(t))]k_i(t) \tag{20}$$

where $\frac{\partial \delta_i}{\partial h_j} > 0, \frac{\partial^2 \delta_i}{\partial h_j^2} < 0, \frac{\partial n_i}{\partial h_j} < 0, \frac{\partial^2 n_i}{\partial h_j^2} < 0$

The Hamiltonian corresponding to each group i problem[2] of choosing its consumption and hostility trajectories can be expressed as:

$$H_i(t) = e^{-\rho_i t} u_i(c_i(t), h_i(t)) + \lambda_i(t)\{f_i(k_i(t)) - c_i(t) - h_i(t) - [\delta_i(h_j(t)) + n_i(h_j(t))]k_i(t)\}$$
$$+\lambda_j(t)\{f_j(k_j(t)) - c_j(t) - h_j(t) - [\delta_j(h_i(t)) + n_j(h_i(t))]k_j(t)\} \tag{21}$$

and the first-order conditions are:

$$\mathrm{e}^{-\rho_i t} u_i'(c_i(t)) - \lambda_i(t) = 0 \tag{22}$$

$$\mathrm{e}^{-\rho_i t} u_i'(h_i(t)) - \lambda_i(t) - \lambda_j(t)\left[\frac{\partial \delta_j}{\partial h_i} + \frac{\partial n_j}{\partial h_i}\right]k_j(t) = 0 \tag{23}$$

$$\dot{\lambda}_i(t) = -\lambda_i(t)\{f_i'(k_i(t)) - [\delta_i(h_j(t)) + n_i(h_j(t))]\} \tag{24}$$

Equation (22) is an optimality condition: for the representative consumer in group i to be in equilibrium, the marginal utility of consumption must equal the marginal utility of wealth (λ_i). In view of this equality, the complementary optimality condition indicated by equation (23) can be interpreted as requiring that the marginal utility of an additional unit of hostile activity must equal the marginal utility of the consumption forgone plus the marginal impact of hostile actions on the net capital-investment possibilities per capita of the rival group. Value is positively associated with scarcity. This association is reflected by the motion equation of the costate variable. Dividing both sides of equation (24) by λ_i, the rate of change of the shadow price of group i's per capita capital is indeed required to decrease with its marginal net accumulation level, which is affected by the hostile actions of group j.

Consistently with assumption 12, let the utility function be:

$$u_i = c_i^{\beta_i} + h_i^{\gamma_i} \tag{25}$$

then the utility-maximizing consumption growth rate of each group i in civil war is given by:

$$\tilde{c}_i^{CW}(t) \equiv \frac{\dot{c}_i(t)}{c_i(t)} = \frac{[f_i'(k_i(t))]_{CW} - (\rho_i + \delta_i(h_j) + n_i(h_j))}{1 - \beta_i} \tag{26}$$

Comparing the *civil-war no-arbitrage rule* of consumption (equation (26)) to the *partition no-arbitrage rule* of consumption (equation (4)), it is interesting to note that when the hostile actions taken by group j are mainly and effectively directed to inflict casualties upon group i rather than to damage its capital stock, the *civil-war* rate of growth of the ith group's consumption is larger than the *partition* rate of growth of its consumption. As a consequence, it also exceeds its federally utility-maximizing rate of consumption growth in a strictly political federation when group i is endowed at time t with a larger initial capital–labor ratio and a lower

[2] That is, maximizinge $\int_0^\infty \mathrm{e}^{-\rho_i t} u_i(c_i(t), h_i(t))\mathrm{d}t$ subject to its and the opponent's capital-motion equations. An extension of this problem that incorporates the group's possible objective of bringing its opponent to submission has been considered by adding a submission-function that depends on the relative size of the rival groups' human and material resources.

population growth rate than its counterpart, and has a rate of time preference that is not considerably larger than that of the imaginary crossbreed. The underlying rationale is as follows.

Recalling Assumption 13, when the actions of group *j* are mainly directed to inflict casualties and are effective (bloody warfare), the capital–labor ratio's depreciation rate for group *i* during the civil-war ($\delta_i(h_j) + n_i(h_j)$) and, consequently, its user-cost of (per capita) capital are smaller than those under partition (i.e. $\rho_i + \delta_i\,(h_j) + n_i\,(h_j) < \rho_i + \delta_i + n_i$). Moreover, starting from the same initial capital–labor ratio and consumption level, per-capita capital accumulation in civil war by group *i* is smaller than that under partition due to diversion of resources to military activities and the damage to capital inflicted by group *j*'s operations. Recalling that the production function is concave, group *i*'s marginal product of capital in the civil war is larger than that under partition. In sum, the difference between group *i*'s marginal-product and user-cost in a casualty-intensive warfare launched by group *j* is larger than that under partition and, in view of equations (4) and (26), facilitating a higher consumption growth rate for group *i*.[3]

Actions taken in a civil war may be aimed at destroying the capital stock of the rival group and hence its production capacity – i.e. sabotage. The impact effect of these actions is a lower consumption for the recipient group due to a lower level of production and diversion of resources to rebuild capital stock. In a sabotage-intensive warfare the user costs for the recipient group are larger than those under partition and a strictly political federation. Despite the decline in its capital stock, the marginal product of per capita capital for the recipient group is not necessarily larger than those under partition and a strictly political federation, due to the disruptions to the production process. It is therefore likely that the rise in the user costs exceeds the rise in the marginal product of capital. This implies that during a sabotage-intensive campaign the initial decrease in the recipient group's level of consumption is likely to be accompanied by a lower consumption growth rate than that under partition. From the perspective of lifetime-utility-maximizing consumption, sabotage is, in this case, a more effectively harming course of action (in particular, for the smaller and poorer group) than bloody warfare.

Attention should also be paid to the evolution of the level of hostility. The growth rate of hostility displayed by group *i* is:

$$\frac{\dot{h}_i(t)}{h_i(t)} = \left[\gamma_i(\gamma_i - 1) - (\delta_j''(h_i) + n_j''(h_i))\lambda_j k_j\right]^{-1}$$
$$\left[(\beta_i - 1)\beta_i c_i^{\beta_i - 2}\dot{c}_i + \left(\delta_j'(h_i) + n_j'(h_i)\right)[\dot{\lambda}_j k_j + \lambda_j \dot{k}_j\right] \tag{27}$$

(see Appendix C).

As the sum in the brackets in the first term on the right-hand side of this expression is negative, group *i*'s rate of growth of hostility toward group *j* increases with the evolution of group's *j* wealth. That is, group *i* becomes more aggressive as group *j*'s capital stock grows along time. This suggests that conflict intensifies as the wealth-gap between two groups, and consequently the poorer group's feelings of relative deprivation, deepens. Indeed, large and increasing wealth-differentials between population groups (classes) have led to the civil wars and revolutions in France, Mexico, Russia, China and Cuba. They also have contributed to the riots of African Americans during the 1960s, to the recent civil-unrests in France, Britain and Holland involving the descendants of the 1970s guest workers, and to the events that brought about the change of regime in the former Rhodesia and in South Africa. This conclusion is also in line with the arguments made by Frey and Luechinger (2003) and Faria and Arce (2005) that

[3] This analytically derived result is consistent with the folk saying: 'Let us eat and drink; for to morrow we shall die' (Bible, Isaiah 22:13).

increasing economic opportunities for the disenfranchised reduce their inclination to be engaged in terrorist activities.

CONCLUSION

The utility-maximizing consumption trajectories associated with possible political states of affairs in dual-population lands were derived and compared by extending Ramsey's generic model of consumption and accumulation. The comparison of the utility-maximizing consumption trajectories may identify the conditions that generate political instability and transition from one political state of affairs to another.

It was shown that, starting from the same initial capital–labor ratio, the federally utility-maximizing consumption growth rate for group i in a strictly political federation is lower than its utility-maximizing consumption growth rate under sole sovereignty in the case of partition if its population growth rate is lower than that of group j, if it is wealthier than group j, and if its rate of time preference is not considerably larger than that of the federal representative imaginary crossbreed. In contrast, the members of group i may economically benefit from joining a federation, even when they are wealthier and multiplying in a lower rate than their counterparts in group j, if the federation facilitates technological transfer and if their initial technology is inferior to the hybrid.

It was also shown that the utility-maximizing growth rate of consumption for a group engaged in a civil war is larger than that attainable under partition, and possibly larger than that prescribed by a strictly political federation, if its adversary's warfare is mainly aimed at inflicting casualties. In contrast, and in addition to an initial drop in consumption, the recipient group's consumption growth rate during a sabotage-intensive warfare is likely to be smaller than those under partition and federation. These results suggest that a sabotage-intensive campaign is likely to be more effective than a casualty-intensive one in affecting the utility of the recipient group's members.

Finally, two comments are made on the limits, and consequently possible extensions, of the analysis (the civil-war one in particular) which, following Ramsey, emphasized the role of consumption in determining people's utility (or happiness) level. The first comment is that the aforementioned possible advantage of a sabotage-intensive campaign is moderated, and may even be inversed, when the recipient group's members suffer from loss of utility due to the cumulative (in the case of long memory) or current (in the case of short memory) number of casualties. The second comment is concerned with terror activities. There has been an escalation in recent years in the use of suicide bombing, remotely controlled car and road-side explosions, homemade-rocket launching and execution of kidnapped civilians in lands populated by two, or more, rival groups. These modes of warfare are adopted by radical groups that are unlikely to be successful in confronting their adversary in an overt manner. In addition to conventional warfare disparity and cultural and ideological factors, the proliferation of such extreme types of warfare can be explained by their ability to spread terror. The analysis of terror requires an expansion of the civil-war model to incorporate the effect of uncertainty about the time and location of the aforementioned actions on the recipient group's members' utility.

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APPENDIX

Appendix A: The Crossbreed's Lifetime Expected Utility

Claim. $J = \int_0^\infty \phi(t) \int_0^t \mathrm{e}^{-\rho\tau} u(\tau)\mathrm{d}\tau \mathrm{d}t = \int_0^\infty \mathrm{e}^{-\rho t} u(t)\Phi(t)\mathrm{d}t$.

Proof. Let $F(t)$ be the cumulative density function associated with the probability of dissolution at t (i.e. the probability of continuation up to t), then:

$$\phi(t) = F'(t) \tag{A1}$$

and equation (9) can be rendered as:

$$J = \int_0^\infty F'(t)\left\{\int_0^t \mathrm{e}^{-\rho\tau} u(\tau)\mathrm{d}\tau\right\}dt = \int_0^\infty v(t)\mathrm{d}U \tag{A2}$$

where

$$v = \int_0^t \mathrm{e}^{-\rho\tau} u(\tau)\mathrm{d}\tau \tag{A3}$$

and

$$U = -(1 - F(t)) \tag{A4}$$

The integration by parts rule suggests that:

$$J = \int_0^\infty v\mathrm{d}U = Uv - \int_0^\infty U\mathrm{d}v \tag{A5}$$

Note, however, that

$$Uv = -\left[(1 - F(t))\int_0^t \mathrm{e}^{-\rho\tau} u(\tau)\mathrm{d}\tau\right]_0^\infty = 0 \tag{A6}$$

because when evaluated at the lower limit

$$Uv = -\left[(1-F(0))\int_0^0 \mathrm{e}^{-\rho\tau}u(\tau)\mathrm{d}\tau\right] = 0 \tag{A7}$$

and when evaluated at the upper limit

$$Uv = -\left[(1-F(T))\int_0^\infty \mathrm{e}^{-\rho\tau}u(\tau)\mathrm{d}\tau\right] = 0 \tag{A8}$$

as

$$\lim_{t\to\infty} F = 1 \tag{A9}$$

Hence

$$J = -\int_0^\infty U\mathrm{d}v \tag{A10}$$

By virtue of equation (A3)

$$\mathrm{d}v = \mathrm{e}^{-\rho\tau}\mathrm{d}\tau \tag{A11}$$

and the substitution of equations (A4) and (A11) into (A10) implies

$$J = \int_0^\infty \mathrm{e}^{-\rho t}u(t)\Phi(t)\mathrm{d}t \tag{A12}$$

where

$$\Phi(t) \equiv -U(t) = 1 - F(t) \tag{A.13}$$

and indicating the probability of the survival of the federation at least until t.

Appendix B: The Crossbreed's Utility-maximizing Consumption Growth Rate

Following Appendix A and substituting equation (10) for c_i, the imaginary crossbreed's life-time utility is:

$$J = \int_0^\infty \mathrm{e}^{-\rho t}\left\{\sum_{i=1}^{2} p_i(t)u_i(\underbrace{f_i(k_i(t)) - (\delta_i + n_i)k_i(t) - \dot{k}_i(t)}_{c_i(t)})\right\}\Phi(k_1(t), k_2(t))\mathrm{d}t \tag{B1}$$

By virtue of Euler's equation, $\dfrac{\partial J}{\partial k_i} - \dfrac{\mathrm{d}}{\mathrm{d}t}\left(\dfrac{\partial J}{\partial \dot{k}_i}\right) = 0$, the necessary condition for maximum life-time utility is:

$$\mathrm{e}^{-\rho t}p_i u_i'(c_i)[f_i'(k_i) - (\delta_i + n_i)]\Phi + \mathrm{e}^{-\rho t}p_i u_i(c_i)\Phi_{k_i} + \frac{\mathrm{d}}{\mathrm{d}t}\left(\mathrm{e}^{-\rho t}p_i u_i'(c_i)\Phi\right) = 0 \tag{B2}$$

which implies:

$$p_i u_i'(c_i)[f_i'(k_i) - (\rho + \delta_i + n_i)]\Phi + p_i u_i(c_i)\Phi_{k_i} + \dot{p}_i u_i'(c_i)\Phi + p_i u_i''(c_i)\Phi \dot{c}_i + p_i u_i'(c_i)\dot{\Phi} = 0 \tag{B3}$$

Divide both sides of equation (B3) by $p_i u_i' \, \Phi$ and solve for $\dot{c}_i$ to obtain

$$\dot{c}_i = \frac{[f_i'(k_i) - (\rho + \delta_i + n_i)] + \left(\frac{u_i}{u_i'}\right)\left(\frac{\Phi_{k_i}}{\Phi}\right) + \left(\frac{\dot{p}_i}{p_i}\right)}{-u_i''(c_i)/u_i'(c_i)} \tag{B4}$$

Note that by virtue of equation (2):

$$\frac{u_i}{u_i'} = c_i / \beta_i \tag{B5}$$

and

$$-\frac{u_i''}{u_i'} = (1 - \beta_i)/\beta_i c_i \tag{B6}$$

Note further that by virtue of equation (5)

$$\frac{\Phi_{k_i}}{\Phi} = -2\mu(k_i - k_j) \tag{B7}$$

By definition:

$$\frac{\dot{p}_i}{p_i} \equiv \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{P_i}{P_i + P_j}\right)}{\frac{P_i}{P_i + P_j}} \tag{B8}$$

and recall equation (1):

$$\frac{\dot{p}_i}{p_i} = (n_i - n_j)p_j \tag{B9}$$

Equation (11) is obtained by substituting equations (B5) – (B9) into equation (B4) and dividing both sides by c_i.

Appendix C: The Utility-maximizing Rate of Change of Hostility

By differentiating:

$$e^{-\rho_i t} u_i'(h_i(t)) - \lambda_i(t) - \lambda_j(t)\left[\frac{\partial \delta_j}{\partial h_i} + \frac{\partial n_j}{\partial h_i}\right] k_j(t) = 0 \tag{C1}$$

with respect to time:

$$e^{-\rho_i t}\left[u_i''(h_i(t))\dot{h}_i(t) - \rho_i u_i'(h_i(t))\right] - \dot{\lambda}_i(t) - \dot{\lambda}_j(t)\left[\frac{\partial \delta_j}{\partial h_i} + \frac{\partial n_j}{\partial h_i}\right] k_j(t) - \dot{k}_j(t)$$
$$\left[\frac{\partial \delta_j}{\partial h_i} + \frac{\partial n_j}{\partial h_i}\right]\lambda_j(t) - \lambda_j(t)\left[\frac{\partial^2 \delta_j}{\partial h_i^2} + \frac{\partial^2 n_j}{\partial h_i^2}\right] k_j(t)\dot{h}_i(t) = 0 \tag{C2}$$

and by rearranging terms, equation (27) is obtained.