J Econ (2008) 94:261–274
DOI 10.1007/s00712-008-0005-3

Persistent high ambition and substance abuse: a rationalization of a vicious circle

Amnon Levy · João Ricardo Faria

Received: 18 June 2007 / Revised: 28 December 2007 / Published online: 6 March 2008

Abstract While the current use of some mind-altering substances alleviates current level of depression, it facilitates future depression. Our analysis incorporates this intertemporal tradeoff and shows that the stationary status of a consistently overly ambitious sophisticated user is improved by impatience, and that this improvement is amplified by the ratio of the instantaneous depression-relief effect to the status-degradation effect of the mind-altering substance. The analysis also shows that the existence of a supportive personal community leads to permanent cyclical use of mind-altering substances.

Keywords Ambition · Failure · Depression · Substance abuse · Relief · Community support · Cycles

JEL Classification D91 · I12

1 Introduction

The possible relationship between mental illness, such as depression, and the demand for mind-altering substances, such as alcohol and narcotic drugs, has been empirically explored by economists. Saffer and Dave (2005) have shown that individuals with a history of mental illness are 26% more likely to consume alcohol and 66% more likely to consume cocaine. To explain this possible relationship psychologists have suggested that a failure to complete a task is profoundly registered in

A. Levy (✉)
School of Economics, University of Wollongong, Wollongong, NSW 2522, Australia
e-mail: amnon_levy@uow.edu.au

J. R. Faria
Nottingham Business School, Nottingham Trent University, Nottingham, UK

the brain and hence leads to depression. Stress generated by a loss of opportunities, partners, parents, siblings, or friends has been frequently associated with the onset of depression, which, in some cases, has led to addiction to mind-altering substances. Changes in the level of various chemicals that transmit nerve impulses in the brain account for depressive state of mind. A low level of such neurotransmitters has been thought to be associated with depression. Alcohol and narcotic drugs affect the sensitivity of the central nervous system, including the brain, in proportion to the amount circulating in the body fluids. To certain individuals, the presence of these substances in the blood produces a sense of elation that they have been unable to achieve in other ways. In time they attempt to maintain this sense of elation by relying on alcohol or narcotic drugs. They fail to recognize that there are emotional needs in their personalities that are being gratified by the use of these substances. (cf. Vaillant 1983; Aseltine et al. 1998; Swendsen and Merikangas 2000; Sitharthan et al. 2001; Langbehn et al. 2006.)

In economics, the most influential theory of consumption of mind-altering substances has been published by Becker and Murphy (1988). The Becker–Murphy rational addiction model has suggested that the consumer's utility from addictive substances depends on the quantity consumed and also on the degree of addiction. Addiction increases as a by-product of consumption, but is gradually depreciated with time. Therefore, current consumption affects future use. In particular, present consumption of the substance can increase its future consumption. Empirical studies on the consumption of light mind-altering substances such as tobacco (Chaloupka 1991; Becker et al. 1994) and coffee (Olekalns and Bardsley 1996) and on the consumption of stronger mind-altering substances such as alcohol (Waters and Sloan 1995; Grossman et al. 1998) and cocaine (Grossman and Chaloupka 1998) have led to results that are consistent with this theory. Furthermore, Dockner and Feichtinger (1993) have shown that the Becker–Murphy model is capable of explaining cyclical consumption paths expressed as damped or explosives waves or limit cycles. Feichtinger and Wirl (1995) have used a similar framework to show that the difference between individual discounting and market interest rates can explain rational cycles as well. The idea of rational cyclical consumption has been adapted by Behrens et al. (2002) to deal with the consumption of mind-altering addictive substances and to explain cycles of drug use. Their findings have been illustrated by numerical analysis based on empirical data from the US cocaine epidemic.[1]

However, the attempts of economists to provide a rationale to the persistent and significant consumption of mind-altering substances with forward-looking utility maximization have ignored the aforementioned possible relationship between the consumption of these substances and depression and the role of these substances in generating instantaneous relief.

Our present paper develops an optimal control model of rational consumption of mind-altering substances that explicitly deals with these aspects. The model incorporates the intertemporal tradeoff associated with the use of mind-altering substances: the

[1] Permanent cycles in alcohol consumption seem to be empirically significant. For instance, Kerr et al. (2002) have suggested the possible existence of subgroups of consumers that move between abstinence and light drinking of alcoholic beverages and moderate and heavy drinking.

alleviation of the user's current depression, on the one hand, and the degradation of the user's status that intensifies his future depression level, on the other hand. Consistent with Maslow's (1954) motivation theory, depression is assumed to be stemming from a difference between actual status and desired status—the greater the desired-actual status differential the stronger the depression.

As in the theory of rational addiction, sophistication is assumed. That is, our analysis considers the case of people who use a mind-altering substance to maximize their lifetime well-being. In particular, we focus on the case of people who are endowed with a long-lasting strong inclination to set their desired status relatively high and use mind-altering substances to moderate their level of depression in a calculated manner. We refer to them as COASUs—consistently overly ambitious sophisticated users. We analyze the properties of the mind-altering-substance-consumption path that minimizes their lifetime depression. Our preliminary analysis suggests that a COASU is impatient: his rate of time preference exceeds the rate of his status-maturation. Yet the COASU's stationary status is improved by impatience and this improvement is amplified by the ratio of the instantaneous depression-relieving effect (the blessing) of the mind-altering substance to the status-eroding effect (the curse) of this substance.

Concern and compassion induce family members, friends and some community organizations to support users. The support may take the form of encouragement to undergo a rehabilitating treatment, or accommodation and improvement of the user's status by extending material and mental assistance and combating stigmatization and marginalization.[2] Empirical studies have stressed the significance of the first form of support (cf., Booth et al. 1992; Aseltine et al. 1998; Marshall et al. 2005). We extend the aforementioned analysis to explore the possible implications of the second form of community support for the COASU's consumption of mind-altering substances. In contrast to the initial analysis, a more complex, cyclical consumption path of the mind-altering substance is possible in the extended framework. Intuitively, the improvement in the COASU's status, following a supportive community effort, moderates the COASU's consumption of the mind-altering substance for a while. However, this moderation in substance abuse lessens the community level of concern, which, in turn, reduces the supportive community effort, lowers the COASU's status and worsens his depression. As the COASU's depression intensifies so does his consumption of the mind-altering substance, which, in turn, increases the community's levels of concern and support, improves the COASU's status, and so forth.

Our analysis is organized as follows. We introduce the set of assumptions portraying the COASU and his vicious circle in Sect. 2. Using this set of assumptions, we derive in Sect. 3 the consumption rule of a mind-altering substance for a lifetime depression-minimizing COASU in the case where there is no external (community) reaction to substance abuse. We compute the COASU's stationary status and substance abuse in this case and display their asymptotic stability properties. In Sect. 4 we introduce a supportive community reaction and spell the conditions that generate a permanent cyclical pattern in the COASU's consumption of mind-altering substances.

[2] It is also possible that other users provide material, mental and social support (cf., Levy et al. 2006).

2 The COASU and his vicious circle

Let $x^*(t) \in \mathbb{R}_+$ denote the COASU's desired status at time t—a combined index of his ambition with regard to material security, inclusion, affection, reputation and self-fulfillment. (cf., Maslow 1954; Schultz 1967). Let $x(t) \in \mathbb{R}$ indicate the COASU's actual status at t, which can be negative when severe material insecurity, exclusion, hatred, bad reputation and/or failure to reach self-fulfillment are experienced. Let $s(t) \in \mathbb{R}_+$ represent the COASU's level of depression at t from not attaining his desired status and $c(t) \in \mathbb{R}_+$ his consumption of the mind-altering substance at t. The following assumptions describe the relationships between COASU's $x^*(t)$, $x(t)$, $s(t)$ and $c(t)$.

Assumption 1 (overly ambitious) $x^*(t) - x(t) > 0$ every t. That is, the COASU sets his desired status too high relatively to his ability and/or organizational and social constraints. He may be meritorious, but his aspired status is extremely difficult to attain. He might also be a victim of marginalization, harassment and abuse based on his racial, ethnic, religious and gender affiliations, appearance, handicap and/or eccentricity.

Assumption 2 (consistently ambitious) $x^*(t) = x^*$ at every t. That is, despite recurrent failures the COASU does not modify his desired status.

Assumption 3 (depression and relief) The COASU's level of instantaneous depression increases with the magnitude of his failure—his desired-actual status differential—and is eased by the numbing effect of his current consumption of the mind-altering substance. More specifically,

$$s(t) = g(c(t))[x^* - x(t)], \tag{1}$$

where $g(c(t))$ is convex, with $g' < 0$, $g'' > 0$, $g(c = 0) = 1$ and $\lim_{c\to\infty} g(c) = 0$, and indicates the depression-relief degree generated by the consumption of the mind-altering substance at t.[3,4,5]

Assumption 4 (status-evolution) The instantaneous change in the COASU's actual status is given by the difference between his status-maturation and his status-degradation

$$\dot{x}(t) = rx(t) - \delta c(t), \tag{2}$$

[3] A broader specification may incorporate a relief-reducing (tolerance) effect of an addictive capital stock A: namely, $g(c; A)$ with $g_c < 0$ and $g_{cA} < 0$ (e.g., $g = A/c$). Our attempts to include another state variable (A) and another state equation (e.g., $\dot{A}(t) = c - \gamma A$, $\gamma > 0$) complicated the analysis enormously; the analysis of the effect of the community support in Sect. 4 in particular (with three state variables—x, A and E), while the addictive aspect is not the focus of our paper.

[4] An alternative quadratic specification, $s(t) = g(c(t))[x^* - x(t)]^2$, has been considered in Appendix A. The quadratic specification satisfies the conditions of Mangasarian's theorem on the sufficiency of Pontryagin's maximum-principle. The consideration of the quadratic specification leads to results and conclusions similar to those obtained in Sect. 3 with the linear specification. We use the linear specification to facilitate the analysis in Sect. 4 of the effects of the supportive community reaction.

[5] Despite his repeated failures, the COASU is not suicidal. He prefers consuming the mind-altering substance to suicide for relieving his depression. A suicidal person is, possibly, too proud to bear the degradation caused by the consumption of mind-altering substances.

where, δ is the COASU's marginal status-degradation caused by the consumption of the mind-altering substance, and r is the rate of gross improvement of the COASU's status—the status-maturation rate. The use of the term status-maturation is consistent with a case where the individual's status and rank in his community, or organization, rise with years of membership and age (i.e., seniority). An alternative specification may be based on the assumption that status-degradation is not independent of, but intensified by, current status: $\dot{x}(t) = [r - \delta c(t)]x(t)$. However, this specification suggests that even a very large consumption of the mind-altering substance does not change status when actual status is zero. Due to this deficiency, the specification given by Eq. (2) is preferred.

Assumption 5 (lifetime depression) A COASU's lifetime depression function is additively separable and reflecting time-consistent preferences, $\int_0^\infty e^{-\rho t} s(t)dt$, with ρ denoting the COASU's rate of time preference.[6]

Assumptions 1–5 complement one another in describing the COASU's vicious circle: failure leads to substance abuse for relieving current depression, substance abuse degrades actual status, and degraded status leads to failure. That is, although substance abuse reduces the COASU's instantaneous level of depression, it raises the potential level of depression for the rest of his life.[7] A COASU cannot escape this vicious circle by changing his physical and social environment. The debilitating effect of substance abuse and his record of failures render him an incompetent and undesired candidate for migration.

3 Depression-minimizing substance consumption

A COASU chooses a consumption path of the mind-altering substance so as to minimize his lifetime depression subject to the evolution of his actual status

$$\underset{c}{\text{Min}} \int_0^\infty e^{-\rho t} g(c(t))[x^* - x(t)]dt; \quad \text{s.t. } \dot{x}(t) = rx(t) - \delta c(t).$$

The current value Hamiltonian corresponding to the above constrained minimization problem is

$$H(t) = g(c(t))[x^* - x(t)] + \lambda(t)[rx(t) - \delta c(t)] \tag{3}$$

[6] An equivalent assumption of additively separable lifetime utility function and time-consistent preferences is used in the rational addiction literature. The case of uncertain life expectancy is analyzed by Levy (2000, 2002).

[7] A negative internality in Herrnstein et al.'s (1993) terminology. Time-inconsistent preferences reflecting a need for immediate gratification (O'Donoghue and Rabin 2000) would exacerbate the COASU's vicious circle.

where the costate variable $\lambda(t)$ indicates the shadow price of the user's status at t. Since $g(c)$ is convex and $x^* - x(t) > 0$, H is convex in the control variable (c). Recalling Assumption 3, H is linearly decreasing in x. In addition to the status motion equation (2), the first-order conditions for minimum lifetime depression are

$$\dot{\lambda}(t) - \rho\lambda(t) = -\frac{\partial H}{\partial x} = g(c(t)) - r\lambda(t) \tag{4}$$

$$\frac{\partial H}{\partial c} = g'(c(t))[x^* - x(t)] - \lambda(t)\delta = 0 \tag{5}$$

and the tranversality condition $\lim_{t\to\infty} H(t)e^{-\rho t} = 0$.

By differentiating equation (5) with respect to time (singular control), substituting the right-hand sides of Eqs. (4) and (5) for $\dot{\lambda}$ and λ, collecting terms and multiplying both sides of the resultant equation by $[1/g'(c(t))]$

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + \delta[g(c(t))/g'(c(t))] + [rx(t) - \delta c(t)]}{[g''(c(t))/g'(c(t))][x^* - x(t)]}. \tag{6}$$

Denoting the elasticity of the instantaneous depression-relief degree with respect to the consumption of the mind-altering substance by

$$\xi(c(t)) \equiv -g'(c(t))\frac{c(t)}{g(c(t))} \tag{7}$$

the COASU's rule of consuming the mind-altering substance can be expressed as

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + rx(t) - \delta c(t)[1 + 1/\xi(c(t))]}{[g''(c(t))/g'(c(t))][x^* - x(t)]}. \tag{8}$$

For tractability, an instantaneous depression-relief degree that displays constant elasticity, $\mu > 0$, and satisfies Assumption 3 is considered

$$g(c(t)) = \exp(-\mu c(t)).^8 \tag{9}$$

With this specification, the COASU's rule of consuming the mind-altering substance is given by

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + (\delta/\mu) + [rx(t) - \delta c(t)]}{-\mu[x^* - x(t)]}. \tag{10}$$

Recalling that $rx(t) - \delta c(t) = \dot{x}(t)$, the substitution of the steady-state condition $(\dot{x} = 0 = \dot{c})$ into Eq. (10) and Eq. (2) implies that the isocline $\dot{c} = 0$ and $\dot{x} = 0$ are,

[8] An alternative specification, which is consistent with Assumption 3 but displaying increasing elasticity in the substance consumption, is $g(t) = 1/[1 + \gamma c(t)]$, where γ is a positive scalar.

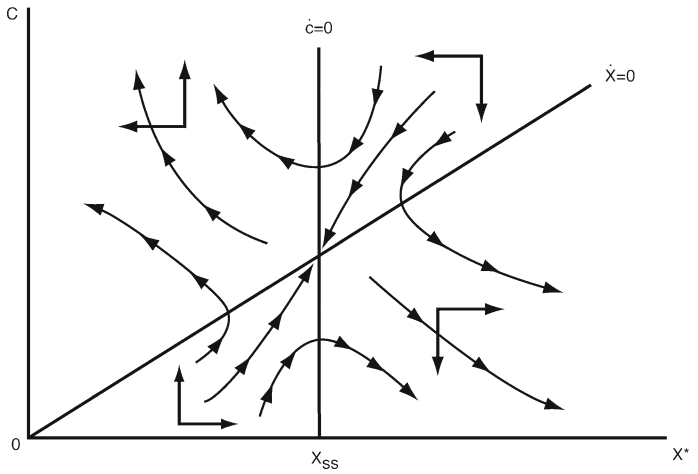

Fig. 1 The phase portrait of status and mind-altering-substance consumption

respectively, given by

$$(\rho - r)[x^* - x_{ss}] - (\delta/\mu) = 0 \tag{11}$$

and

$$c = (r/\delta)x. \tag{12}$$

Subsequently, the COASU's stationary status and substance consumption are

$$x_{ss} = x^* - \frac{\delta/\mu}{\rho - r} \tag{13}$$

and

$$c_{ss} = (r/\delta)\{x^* - [r/\mu(\rho - r)]\}. \tag{14}$$

The nature of the COASU's steady state is identified by computing the eigenvalues of the Jacobian matrix of the system of equations (2) and (10). At the steady state this Jacobian matrix is

$$J \equiv \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial c} \\ \frac{\partial \dot{c}}{\partial x} & \frac{\partial \dot{c}}{\partial c} \end{bmatrix}_{\substack{\dot{x}=0 \\ \dot{c}=0}} = \begin{bmatrix} r & -\delta \\ \rho(r-\rho)/\delta & \rho - r \end{bmatrix}. \tag{15}$$

Since $tr\, J = \rho$ and $\det J = -(r-\rho)^2$, the eigenvalues $\psi_1 = 0.5[\rho + \sqrt{\rho^2 + 4(r-\rho)^2}]$ and $\psi_2 = 0.5[\rho - \sqrt{\rho^2 + 4(r-\rho)^2}]$ have different signs: $\psi_1 > 0$ whereas $\psi_2 < 0$. Hence, as displayed in Fig. 1, the COASU's steady state, (x_{ss}, c_{ss}), is a saddle point.

Along the lower branch of the stable manifold the COASU's status is improved, despite the increase in his consumption of the mind-altering substance; whereas along the upper branch of the stable manifold the COASU's status is degraded despite the moderation of his consumption of the mind-altering substance.

Proposition 1 (impatience) *The COASU's rate of time preference exceeds his rate of status-maturation,* $\rho > r$.

Proof Recalling Eq. (13), a $\rho \leq r$ implies that $x_{ss} \geq x^*$. However, $x_{ss} \geq x^*$ violates Assumption 1. Hence, $\rho > r$ in the case of a COASU.

The steady state comparative statics lead to the following conclusions. As can be seen from Eq. (13), the gap between the COASU's desired status and stationary status is narrowed by the difference between his rate of time-preference and his rate of status-maturation. This narrowing of the desired-stationary status-gap is strengthened by the ratio of the elasticity of the instantaneous depression-relief degree (the blessing) to the status-degradation effect (the curse) of the mind-altering substance. Equation (14) indicates that the COASU's stationary consumption of the mind-altering substance is intensified by the difference between his rate of time preference and his rate of status-maturation and by the elasticity of the instantaneous depression-relief degree provided by the substance, but moderated by the status-degradation effect of the substance. It also reveals that the COASU's stationary consumption of the mind-altering substance is moderated by his rate of status-maturation if $\frac{\delta\rho}{\mu(\rho-r)^2} > x^*$, but intensified by his rate of status-maturation if $\frac{\delta\rho}{\mu(\rho-r)^2} < x^*$. That is, when the COASU's status-degradation coefficient is large (small) and his elasticity of the instantaneous depression-relief degree of the substance as well as his difference between the rate of time preference and rate of status-maturation are small (large), it is likely that a rise in the COASU's rate of status-maturation induces him to moderate (increase) his consumption of the mind-altering substance.

4 Community support and cyclical consumption

Concerns and compassion induce family members, friends and other community members to accommodate substance-use, to materially and mentally support users, and to combat stigmatization and marginalization of users. We assume that the presence of community support modifies the COASU's status motion equation (2) in the following way

$$\dot{x}(t) = r\,x(t) - \delta\,c(t) + \beta\,E(t), \quad \beta > 0, \tag{16}$$

where the current effort $E(t)$ invested by the community in improving the user's status is multiplied by a coefficient β that indicates the community effort's marginal status-improving effect. Due to increasing potential private and public costs, the supportive community effort is assumed to rise with the excessive consumption of the mind-altering substance

$$\dot{E}(t) = \alpha[\,c(t) - \hat{c}], \quad \beta > 0, \;\; \hat{c} \geq 0 \tag{17}$$

where α indicates the community inclination to react and $\hat{c}$ the consumption of the mind-altering substance perceived by the community as the upper bound of normal use.[9]

Although their objective is to improve users' status, community care and support do not necessarily enhance the COASU's status. The COASU anticipates the community support and incorporates it into his selection process of the depression-relieving consumption path of the mind-altering substance. That is, the COASU chooses c so as to minimize $\int_0^\infty e^{-\rho t} s(t) dt$ subject to his actual status-evolution equation (16) and the supportive-community-effort-investment equation (17). The current value Hamiltonian associated with this intertemporal decision problem is

$$H(t) = g(c(t))[x^* - x(t)] + \lambda(t)[rx(t) - \delta c(t) + \beta E(t)] + \theta(t)\,\alpha[c(t) - \hat{c}] \tag{18}$$

where the costate variable $\theta(t)$ indicates the shadow price of the supportive community reaction at t. The first-order conditions are

$$\frac{\partial H}{\partial c} = g'(c(t))[x^* - x(t)] - \lambda(t)\delta + \theta(t)\,\alpha = 0 \tag{19}$$

$$\dot{\lambda}(t) - \rho\lambda(t) = -\frac{\partial H}{\partial x} = g(c(t)) - \lambda(t)r \tag{20}$$

$$\dot{\theta}(t) - \rho\theta(t) = -\frac{\partial H}{\partial E} = -\lambda(t)\beta \tag{21}$$

and the tranversality condition is $\lim_{t\to\infty} H(t)e^{-\rho t} = 0$.

In the present case, the first-order conditions allow a complex dynamics of the mind-altering-substance consumption such as persistent cyclical paths. A possible long-run equilibrium in this community-reaction augmented model is an invariant manifold—a limit cycle.[10] In order to examine the possibility of such a complex dynamics we use Eq. (19) to express the consumption of the mind-altering substance as a function of λ, θ and x:

$$\{g'(c(t))[x^* - x(t)] = \lambda(t)\delta - \theta(t)\,\alpha\} \Rightarrow \{c(t) = c(x(t),\ \lambda(t),\ \theta(t))\}. \tag{19'}$$

By differentiation, we obtain the following properties of the COASU's substance-consumption function:

[9] The consumption level of a mind-altering substance perceived as normal is culture-dependent. An acceptable level of alcohol consumption in one culture might be considered alcoholism in another (Vaillant 1983).

[10] As showed by Dockner and Feichtinger (1993) and indicated in the introduction, the theory of rational addiction is capable of explaining cyclical consumption paths. However, the mechanism of adjacent and distant complementarity used in their paper is not capable to explain the cycle derived here. In Feichtinger and Wirl (1995) the soft budget constraint plays a similar role to the supportive community in our model. We thank the referee for making this point.

$$c_x = \frac{g'(c(t))}{g''(c(t))[x^* - x(t)]} < 0, \quad c_\lambda = \frac{\delta}{g''(c(t))[x^* - x(t)]} > 0,$$
$$c_\theta = \frac{-\alpha}{g''(c(t))[x^* - x(t)]} < 0. \tag{22}$$

The substitution of Eq. (19′) into Eqs. (16), (17), (20) and (21) yields:

$$\dot{x}(t) = r\,x(t) - \delta\,c(x(t),\,\lambda(t),\,\theta(t)) + \beta\,E(t) \tag{16′}$$

$$\dot{E}(t) = \alpha c(x(t),\,\lambda(t),\,\theta(t)) - \alpha\hat{c} \tag{17′}$$

$$\dot{\lambda}(t) = [\rho - r]\lambda(t) + g(c(x(t),\,\lambda(t),\,\theta(t))) \tag{20′}$$

$$\dot{\theta}(t) = \rho\theta(t) - \lambda(t)\beta. \tag{21′}$$

With this presentation of the first-order conditions, the condition for a limit cycle between the COASU's status and the supportive community reaction; and, consequently [through Eq. (19′)], for a cyclical consumption of the mind-altering substance; can be summarized as follows.

Proposition 2 *$\rho > r$ for a COASU, and this inequality and $r\,\delta < 2\alpha\beta$ are the necessary conditions for the existence of a limit cycle between his status (x) and the supportive community effort (E). (See Appendix B for proof.)*

Corollary *If the COASU's status (x) displays cyclical behavior, his mind-altering-substance consumption (c) is cyclical. (See Appendix B for proof.)*

For a COASU, $\rho > r$. Therefore the existence of a permanent cyclical consumption path of the mind-altering substance is conditioned on the inequality $r\,\delta < 2\alpha\beta$. According to this inequality a limit cycle exists as long as the COASU's rate of status-maturation r does not exceed the ratio of the reactive-support effect $\alpha\beta$ to the status-degradation effect of the mind-altering substance rate δ (i.e., $r < \alpha\beta/\delta$). As a consequence, a sufficient condition for the existence of the limit cycle would be that the COASU's rate of time preference ρ does not exceed the ratio of the reactive-support effect $\alpha\beta$ to the status-degradation effect of the mind-altering substance δ (i.e., $\rho < \alpha\beta/\delta$).

The cycle indicated in Proposition 2 and the corollary is interpreted as follows. A rise in the COASU's consumption of the mind-altering substance motivates his community to increase its support, which leads to an improvement in his status, which leads to a reduction in his use of the mind-altering substance, which further improves his status. However, the reduction in the COASU's use of the mind-altering substance moderates his community support, which lowers his status, which increases the use of the mind-altering substance, which decreases the COASU's status. The increase of the COASU's consumption of the mind-altering substance motivates his community to increase support, and the cycle is repeated.

5 Concluding remarks

This paper analyzes the vicious circle of depression and substance abuse in the case of a consistently overly ambitious sophisticated user who selects a mind-altering-substance-consumption path to minimize the sum of the discounted instantaneous levels of depression stemming from the gap between his desired status and actual status over the rest of his lifespan. The vicious circle reflects the negative internality associated with substance abuse: present consumption of the mind-altering substance alleviates the present level of depression, but leads to future failures and, consequently, depression. The preliminary analysis shows that the COASU's stationary status is improved by the difference between his rate of time preference and his rate of status-maturation, and that this improvement is amplified by the ratio of the effectiveness of the substance in reducing instantaneous depression (the blessing) to the status-degradation effect of the substance (the curse). When a supportive community reaction to excessive use is taken into account, the COASU's mind-altering-substance consumption displays permanent cyclical paths.

Appendix A: Quadratic instantaneous-depression function

The alternative quadratic specification of the instantaneous-depression function indicated in footnote 5 makes the Hamiltonian convex in the state variable x:

$$H(t) = g(c(t))[x^* - x(t)]^2 + \lambda(t)[rx(t) - \delta c(t)]. \tag{A1}$$

In this case, the conditions of Mangasarian's theorem on the sufficiency of Pontryagin's maximum-principle are satisfied. In addition to the state equation (2), the conditions for minimum lifetime suffering are

$$\dot{\lambda}(t) - \rho\lambda(t) = -\frac{\partial H}{\partial x} = 2g(c(t))[x^* - x(t)] - \lambda(t)r \tag{A2}$$

$$\frac{\partial H}{\partial c} = g'(c(t))[x^* - x(t)]^2 - \lambda(t)\delta = 0 \tag{A3}$$

and the tranversality condition $\lim_{t\to\infty} \lambda(t)x(t) = 0$.

By differentiating Eq. (A3) with respect to time, substituting the right-hand sides of Eqs. (A2) and (A3) for $\dot{\lambda}$ and λ, collecting terms and multiplying both sides of the resultant equation by $[1/g'(c(t))]$ we obtain

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] + 2\delta[g(c(t))/g'(c(t))] + 2\overbrace{[rx(t) - \delta c(t)]}^{\dot{x}(t)}}{[g''(c(t))/g'(c(t))][x^* - x(t)]}. \tag{A4}$$

Recalling the constant-elasticity specification of the instantaneous depression-relieving function, then

$$\dot{c}(t) = \frac{(\rho - r)[x^* - x(t)] - 2(\delta/\mu) + 2\overbrace{[rx(t) - \delta c(t)]}^{\dot{x}(t)}}{-\mu[x^* - x(t)]}. \tag{A5}$$

By substituting the steady-state condition ($\dot{x} = 0 = \dot{c}$) into Eq. (A5) the isocline $\dot{c} = 0$ is given by

$$(\rho - r)[x^* - x_{ss}] - 2(\delta/\mu) = 0, \tag{A6}$$

and the stationary status and substance consumption are

$$x_{ss} = x^* - 2\left[\frac{\delta}{\mu(\rho - r)}\right] \tag{A7}$$

and

$$c_{ss} = \left(\frac{r}{\delta}\right) x^* - 2\left[\frac{r}{\mu(\rho - r)}\right]. \tag{A8}$$

Appendix B: Proofs of Proposition 2 and Corollary

Proof of Proposition 2: Following Feichtinger et al. (1994), it is necessary to show, for proving the existence of a limit cycle, that the determinant of the Jacobian matrix of the system of equations (16′), (17′), (20′) and (21′)

$$|J| = \begin{vmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial E} & \frac{\partial \dot{x}}{\partial \lambda} & \frac{\partial \dot{x}}{\partial \theta} \\ \frac{\partial \dot{E}}{\partial x} & \frac{\partial \dot{E}}{\partial E} & \frac{\partial \dot{E}}{\partial \lambda} & \frac{\partial \dot{E}}{\partial \theta} \\ \frac{\partial \dot{\lambda}}{\partial x} & \frac{\partial \dot{\lambda}}{\partial E} & \frac{\partial \dot{\lambda}}{\partial \lambda} & \frac{\partial \dot{\lambda}}{\partial \theta} \\ \frac{\partial \dot{\theta}}{\partial x} & \frac{\partial \dot{\theta}}{\partial E} & \frac{\partial \dot{\theta}}{\partial \lambda} & \frac{\partial \dot{\theta}}{\partial \theta} \end{vmatrix} \tag{B1}$$

and the term

$$\mathrm{M} = \begin{vmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial x} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{E}}{\partial E} & \frac{\partial \dot{E}}{\partial \theta} \\ \frac{\partial \dot{\theta}}{\partial E} & \frac{\partial \dot{\theta}}{\partial \theta} \end{vmatrix} + 2 \begin{vmatrix} \frac{\partial \dot{x}}{\partial E} & \frac{\partial \dot{x}}{\partial \theta} \\ \frac{\partial \dot{\lambda}}{\partial E} & \frac{\partial \dot{\lambda}}{\partial \theta} \end{vmatrix} \tag{B2}$$

are positive when calculated with the steady-state levels ($\overline{x}$, $\overline{E}$, $\overline{\lambda}$, $\overline{\theta}$).[11] In addition, the value of the bifurcation parameter (ρ) given by the following condition

$$|J| = \left(\frac{M}{2}\right)^2 + \rho^2 \left(\frac{M}{2}\right) \tag{B3}$$

must be positive as well. Note that

$$\{|J| = -\beta\alpha\rho\, c_x\,(\rho - r) > 0\} \Leftrightarrow \{\rho > r\} \tag{B4}$$

and

$$\{M = (r - \delta c_x)(\rho - r) + g'[r c_\lambda + 2\beta c_\theta] > 0\} \Leftrightarrow \{r\delta < 2\alpha\beta\}. \tag{B5}$$

From B4, B5 and B3, the necessary conditions for the existence of a limit cycle between x and E are: i. $\rho > r$, and ii. $r\,\delta < 2\alpha\beta$.

Proof of Corollary: The cyclical behavior of c is implied by Eq. (18') when x displays cyclical behavior.

References

Aseltine RH, Gore S, Colten ME (1998) The co-occurrence of depression and substance abuse in late adolescence. Dev Psychopathol 10(3):549–570

Becker GS, Murphy KM (1988) A theory of rational addiction. J Polit Econ August: 675–700

Becker GS, Grossman M, Murphy KM (1994) An empirical analysis of cigarette addiction. Am Econ Rev June: 396–418

Behrens DA, Caulkins JP, Tragler G, Feichtinger G (2002) Why present-oriented societies undergo cycles of drug epidemics. J Econ Dyn Control 26(6):919–936

Booth BM, Russell DW, Soucek S, Laughlin PR (1992) Social support and outcome of alcoholism treatment: an exploratory analysis. Am J Drug Alcohol Abuse 18:87–101

Chaloupka F (1991) Rational addictive behavior and cigarette smoking. J Polit Econ August: 722–742

Dockner EJ, Feichtinger G (1993) Cyclical consumption patterns and rational addiction. Am Econ Rev March: 256–263

Feichtinger G, Wirl F (1995) Persistent cyclical consumption. Ration Soc 7(2):156–166

Feichtinger G, Novak A, Wirl F (1994) Limit cycles in intertemporal adjustment models. J Econ Dyn Control March: 353–380

Grossman M, Chaloupka F (1998) The demand for cocaine by young adults: a rational addiction approach. J Health Econ 17(4):427–474

Grossman M, Chaloupka F, Sirtalan I (1998) An empirical analysis of alcohol addiction: results from the monitoring the future panels. Econ Inq January: 39–48

Herrnstein RJ, Loewenstein G, Prelec D, Vaugham W Jr (1993) Utility maximization and melioration: internalities in individual choice. J Behav Decis Mak 6:149–185

Kerr WC, Fillmore KM, Bostrom A (2002) Stability of alcohol consumption over time: Evidence from three longitudinal surveys from the United States. J Stud Alcohol May: 325–334

Langbehn DR, Philibert R, Caspers KM, Yucuis R, Cadoret RJ (2006) Association of a D2S2944 allele with depression specifically among those with substance abuse or antisocial personality. Drug Alcohol Depend 83(1):33–41

[11] The steady state equilibrium is found when $\dot{\lambda} = \dot{\theta} = \dot{x} = \dot{E} = 0$.

Levy A (2000) Would a rational Lucy take-off without assessing the probability of a crash landing? East Econ J 26(4):431–437
Levy A (2002) Rational eating: can it lead to overweightness or underweightness? J Health Econ 21(5): 887–899
Levy A, Neri F, Grass D (2006) Macroeconomic aspects of substance abuse: diffusion, productivity and optimal control. Macroecon Dyn 10(2):145–164
Marshall JP, Kimball TG, Shumway ST, Miller MM, Jeffries V, Arredondo R (2005) Outcomes of a structured family group in an outpatient alcohol/other drug treatment setting. Alcohol Treat Q 23(4):39–53
Maslow AH (1954) Motivation and personality. Harper & Brothers, New York
O'Donoghue T, Rabin M (2000) The economics of immediate gratification. J Behav Decis Mak 13(2):233–250
Olekalns N, Bardsley P (1996) Rational addiction to caffeine: an analysis of coffee consumption. J Polit Econ October: 1100–1104
Saffer H, Dave D (2005) Mental illness and the demand for alcohol, cocaine, and cigarettes. Econ Inq 43(2):229–246
Schultz W (1967) Joy: expanding human awareness. Grove Press, New York
Sitharthan G, Hough MJ, Sithartan T, Kavanagh DJ (2001) The alcohol helplessness scale and its prediction of depression among problem drinkers. J Clin Psychol December: 1445–1458
Swendsen JD, Merikangas KR (2000) The comorbidity of depression and substance use disorders. Clin Psychol Rev 20:173–189
Vaillant GE (1983) The natural history of alcoholism: causes, patterns, and paths to recovery. Harvard University Press, Cambridge
Waters TM, Sloan FA (1995) Why do People drink? Tests of the rational addiction model. Appl Econ August: 727–736