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GROWTH AND STABILITY IN A MODEL WITH PASINETTIAN SAVING BEHAVIOUR AND NEOCLASSICAL TECHNOLOGY*

by

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We analyse a Kaldor–Pasinetti two-class model of growth and distribution in which fiscal activity is explicitly introduced along the lines of Pasinetti ('Ricardian Debt/Taxation Equivalence in the Kaldor Theory of Profits and Income Distribution', *Cambridge Journal of Economics*, Vol. 13 (1989), pp. 25–36). Following the approach of Darity ('A Simple Analytics of Neo-Ricardian Growth and Distribution', *American Economic Review*, Vol. 71 (1981), pp. 978–993) the model is reduced to a dynamic system where the Cambridge equation is one of the possible steady-state solutions. The conditions for its local stability are studied and a numerical example is presented. The anti-dual case is more likely to occur in order to guarantee the local stability of the Cambridge equation.

1 Introduction

We examine the local stability of the Cambridge result when government taxation as well as expenditures are explicitly considered along the lines of Pasinetti (1989). Recent literature has ignored the issue of the stability of these models. The reason for this is that, in general, it is not possible to analyse stability without considering a proper dynamic model.

The general framework of the paper is the Kaldor–Pasinetti process, a two-class model of growth and distribution. We follow Pasinetti (1989) in dealing with propensities to save corrected by taxation and government savings. However, other aspects of government activities, such as transfer payments and ownership of public enterprises, are not considered in our analysis. Our aim is to extend the Darity (1981) approach to the case when both direct and indirect taxes and an unbalanced government budget are present in a closed economy. In order to study local stability we assume

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that technology can be described by a well-behaved neoclassical production function.

The conditions for local stability of this model can be analysed if the original system of equations is reduced to a smaller dimension. Therefore, we introduce some simplified assumptions following the procedure outlined by Darity (1981). Following this procedure, Pasinetti's model with taxation is reduced to a system of two differential equations: one for the total physical capital and the other for the capital stock owned by capitalists. This compact system summarizes all the relevant information needed. It is shown that one of the possible steady-state solutions of this dynamic system is the Cambridge equation. This means that, despite methodological controversies, our simplifications do not affect the expected result. On the contrary, our approach is an interesting way to discuss stability in this type of model.

There are essential methodological differences between the neoclassical theory of marginal productivity and the post-Keynesian approach to growth and distribution. In this sense our model, dealing with both approaches, may be considered a heterodox model. Nevertheless, this methodological dispute goes beyond the scope of the present work. Samuelson and Modigliani (1966), Baranzini (1975), Ramanathan (1976), Darity (1981) and O'Connell (1995), among others, have attempted this combination of paradigms.

The paper is organized as follows. Section 2 presents the structural form of the Pasinetti (1989) model. In Section 3 the model is simplified and rewritten as a dynamic system where the Cambridge equation is one of the steady-state solutions. The local stability conditions are discussed and a numerical example is presented. Section 4 concludes.

2 Post-Keynesian Saving Propensities

Pasinetti (1989) pointed out that economists have ignored the role of government taxation and expenditure when dealing with the Kaldorian-type models of growth and distribution. To correct this omission Pasinetti has explored alternative budget constraints (equilibrium, surplus and deficit) in a model in which both direct and indirect taxes are contemplated. His model does not incorporate public capital. The notation is standard (see Appendix A) and the structural form of his model is

$$Y \equiv C + I + G \equiv W + P \tag{2.1}$$

$$P = P_{\mathrm{W}} + P_{\mathrm{C}} \tag{2.2}$$

$$G = (1 - s_{\mathrm{g}})T \tag{2.3}$$

$$T = t_W W + t_P(P_W + P_C) + t_i\{(1 - s_W)[(1 - t_W)W + (1 - t_P)P_W] + (1 - s_C)(1 - t_P)P_C + G\} \tag{2.4}$$

$$S = S_W + S_C + S_g \tag{2.5}$$

$$S_W = s_W[(1 - t_W)W + (1 - t_P)P_W] \tag{2.6}$$

$$S_C = s_C(1 - t_P)P_C \tag{2.7}$$

$$S_g = s_g T \tag{2.8}$$

The usual assumptions are $0 < t_W < t_P < 1$, $0 \leq s_W < s_C < 1$ and $s_g > s_W$.[1]

Notice that Dalziel (1989), contrary to Pasinetti (1989), prefers to introduce explicitly P_g (government profit) and K_g (public owned capital, not public enterprises) because, if $s_g \neq 0$ permanently, P_g and K_g will be negative or positive depending on the budget state (deficit or surplus). Pasinetti prefers to deal implicitly with such disequilibrium—this is incorporated in the saving propensities corrected by the effects of government taxation and expenditures. The two alternative approaches are neither conflicting nor present theoretical advantages. However, for mathematical simplicity, Pasinetti's approach is used here.

Substituting (2.3) into (2.4) yields the total taxation function (2.9), and substituting (2.9) into (2.8) yields the government saving function (2.10):

$$T = \beta\{t_W W + t_P P_W + t_P P_C + t_i[(1 - s_W)(1 - t_W)W + (1 - s_W)(1 - t_P)P_W + (1 - s_C)(1 - t_P)P_C]\} \tag{2.9}$$

$$S_g = s_g\beta\{t_W W + t_P P_W + t_P P_C + t_i[(1 - s_W)(1 - t_W)W + (1 - s_W)(1 - t_P)P_W + (1 - s_C)(1 - t_P)P_C]\} \tag{2.10}$$

where $\beta = [1 - t_i(1 - s_g)]^{-1}$ is the correction factor due to the fact that the government also taxes its own expenditure.[2]

Substituting (2.6), (2.7) and (2.10) into (2.5) we have

$$S = s'_{WW} W + s'_{WC} P_W + s'_C P_C \tag{2.11}$$

where

$$s'_{WW} = s_W(1 - t_W) + s_g\beta[t_W + t_i(1 - s_W)(1 - t_W)]$$
$$s'_{WC} = s_W(1 - t_P) + s_g\beta[t_P + t_i(1 - s_W)(1 - t_P)]$$
$$s'_C = s_C(1 - t_P) + s_g\beta[t_P + t_i(1 - s_g)(1 - t_P)]$$

Notice that Pasinetti has assumed that $t_W < t_P$ with the implication

[1]This assumption depends on $G < T$, i.e. the government expenditure needs to be less than taxation, and marginal propensity to consume by the workers must be greater than that of the government; this assumption is more restrictive than that by Pasinetti (1989).

[2]This is not a fundamental assumption; notice that if $t_i = 0$ then $\beta = 1$, which does not affect any of our further results.

that $s'_{WW} > s'_{WC}$. Here we are assuming the opposite case, $t_W > t_P$, and thus $s'_{WC} > s'_{WW}$. It is not difficult to show that $s'_C > s'_{WC} > s'_{WW}$ and $s'_{WW} > s'_C - s'_{WC}$ if $0 < s_W < s_C < 1 \leq 2s_W$. Notice that this assumption does not affect the solution of the model, which is the Cambridge equation. However, with the reversed inequality ($s'_{WW} > s'_{WC}$ instead of $s'_{WC} > s'_{WW}$) it is difficult to sustain local stability, as can be seen in the next section.

As shown by Pasinetti (1989, pp. 27–32), the extended Cambridge equation $r = g/s'_C$ follows from the expressions above and using the standard Pasinettian assumptions: (i) savings proportional to capital for both capitalists and non-capitalists; (ii) the investments equal savings (*ex ante*) equilibrium condition; and (iii) a uniform equilibrium rate of profit for the economy as a whole. This result means that workers' propensity to save does not play any role in determining the rate of profit of the economy. In his 1989 paper, Pasinetti does not deal with stability.[3] In the next section the above system is rewritten and reformulated in a dynamic system following the approach developed by Darity (1981).

3 Existence and Local Stability of Equilibrium

We follow Darity (1981) in dealing with the long-run equilibrium and its local stability with the provision that fiscal policy is now considered. In this sense we extend the Kaldor–Pasinetti model by incorporating a public sector in fiscal activity. The notation is standard (see Appendix A).

Assuming that all savings are invested we have

$$S = \dot{K} = \dot{K}_W + \dot{K}_C = s'_{WW} W + s'_{WC} P_W + s'_C P_C \tag{3.1}$$

where

$$\dot{K}_W = s'_{WW} W + s'_{WC} P_W \tag{3.2}$$

$$\dot{K}_C = s'_C P_C \tag{3.3}$$

We can define the following terms:

$$\dot{L}/L = g \tag{3.4}$$

$$k = K/L \tag{3.5}$$

$$K = K_C + K_W \Rightarrow k_C + k_W = 1 \tag{3.6}$$

$$\dot{k}_C = -\dot{k}_W \tag{3.7}$$

We are able to express (3.1) in per-worker terms, and (3.2) and (3.3) in per-share terms:

$$\frac{\dot{k}}{k} = \frac{\dot{K}}{K} - \frac{\dot{L}}{L} = s'_{WW}\frac{W}{K} + s'_{WC}\frac{P_W}{K} + s'_C\frac{P_C}{K} - g \tag{3.8}$$

[3]See Teixeira and Araújo (1991).

$$\frac{\dot{k}_W}{k_W} = \frac{\dot{K}_W}{K_W} - \frac{\dot{K}}{K} = s'_{WW}\frac{W}{K_W} + s'_{WC}\frac{P_W}{K_W} - s'_{WW}\frac{W}{K} - s'_{WC}\frac{P_W}{K} - s'_C\frac{P_C}{K}$$
$$= s'_{WW}\left(\frac{W}{K_W} - \frac{W}{K}\right) + s'_{WC}\left(\frac{P_W}{K_W} - \frac{P_W}{K}\right) - s'_C\frac{P_C}{K} \quad (3.9)$$

$$\frac{\dot{k}_C}{k_C} = \frac{\dot{K}_C}{K_C} - \frac{\dot{K}}{K} = s'_C\left(\frac{P_C}{K_C} - \frac{P_C}{K}\right) - s'_{WW}\frac{W}{K} - s'_{WC}\frac{P_W}{K} \quad (3.10)$$

Naturally, in view of expression (3.7), either equation (3.9) or equation (3.10) is redundant. With the elimination of (3.9), the above system of three differential equations is reduced to the following system:

$$\dot{k} = k\left(s'_{WW}\frac{W}{K} + s'_{WC}\frac{P_W}{K} + s'_C\frac{P_C}{K} - g\right) \quad (3.11)$$

$$\dot{k}_C = k_C\left[s'_C\left(\frac{P_C}{K_C} - \frac{P_C}{K}\right) - s'_{WW}\frac{W}{K} - s'_{WC}\frac{P_W}{K}\right] \quad (3.12)$$

Taking into consideration that $W = Y - rK$, $P_W = rK_W$ and $P_C = rK_C$, where r stands for the rate of profit, equations (3.11) and (3.12) become

$$\dot{k} = k\left[\left(\frac{Y}{K} - r\right)s'_{WW} + s'_{WC}\frac{rK_W}{K} + s'_C\frac{rK_C}{K} - g\right] \quad (3.13)$$

$$\dot{k} = k_C\left[s'_C\left(r - r\frac{K_C}{K}\right) - s'_{WW}\left(\frac{Y}{K} - r\right) - s'_{WC}r\frac{K_W}{K}\right] \quad (3.14)$$

For convenience we set $k_W = K_W/K$, $k_C = K_C/K$ and use the neoclassical aggregated production function to relate output to capital and labour: $Y = Y(K, N)$, where $Y(.)$ is the production function and N is the labour force actually employed. Assuming that the production function presents constant returns to scale, $Y/L = Y(k, n)$, where $k = K/L$ and $n = N/L$. Defining the relation of output to capital as $Y/K = a = a(K, N)$, $a_K < 0$, $a_N > 0$, $a > Y_K$ if $0 < K < \infty$, the foregoing system, formed by equations (3.13) and (3.14), is then equivalent to

$$\dot{k} = k\{[a(K, N) - r]s'_{WW} + s'_{WC}rk_W + s'_C rk_C - g\} \quad (3.15)$$

$$\dot{k}_C = k_C\{s'_C(r - rk_C) - s'_{WW}[a(K, N) - r] - s'_{WC}rk_W\} \quad (3.16)$$

Inspection of this system reveals four unknowns, k, k_C, n and r. However, neoclassical growth theorists frequently assume permanent full employment ($N = L$), which implies that $n = 1$, and one unknown is eliminated. In the same vein we eliminate r by equating it to the marginal product of capital ($r = Y_K$). Thus, by implication, the model also sets the real wage equal to the marginal product of labour. Then, with $k_W = 1 - k_C$, the system can be written as

$$\dot{k} = k\{s'_{WW}[a - Y_K] + s'_{WC}Y_K(1 - k_C) + s'_C Y_K k_C - g\} \tag{3.17}$$

$$\dot{k}_C = k_C\{s'_C(Y_K - k_C Y_K) - s'_{WW}[a - Y_K] - s'_{WC}Y_K(1 - k_C)\} \tag{3.18}$$

For $\dot{k} = \dot{k}_C = 0$ we have the following steady-state solutions:

$$k_C^* = \frac{s'_{WW}[a(k^*) - Y_K(k^*)] + s'_{WC}Y_K(k^*) - g}{(s'_{WC} - s'_C)Y_K(k^*)} \qquad \text{for } \dot{k} = 0 \tag{3.19}$$

$$k_C^* = 1 - \frac{s'_{WW}[a(k^*) - Y_K(k^*)]}{(s'_C - s'_{WC})Y_K(k^*)} \qquad \text{for } \dot{k}_C = 0 \tag{3.20}$$

Finally, equating (3.19) to (3.20) yields

$$r = Y_K(k^*) = g/s'_C \tag{3.21}$$

Equation (3.21) is the Cambridge equation. It can be seen that the Cambridge equation is one of the possible steady-state solutions of the dynamic system formed by equations (3.17) and (3.18).

Another possible steady-state solution occurs when $k_C = 0$. By equation (3.18) this implies that $\dot{k}_C = 0$, and by equation (3.17), in the steady state ($\dot{k} = 0$), we have

$$a = \frac{g - s'_{WC}r}{s'_{WW}} + r$$

Notice that this equilibrium in the absence of government, $t_W = t_P = s_g = 0$ (see equation (2.11)), corresponds to $Y/K = a = g/s_W$. This result is known as the dual equilibrium or Meade–Samuelson–Modigliani result (MSM land as suggested by Harcourt (1972, p. 222)), and is also called the anti-Pasinetti result or 'euthanasia' of the capitalists. Samuelson and Modigliani (1966) have shown that, if the savings propensity of workers is high enough, this class ends up doing all the accumulation and the capitalists share of total wealth approaches zero. In this case the share of workers' capital in total capital tends asymptotically to unity and the capitalists become irrelevant.[4]

Darity (1981) draws attention to the possibility of an anti-dual outcome (where capitalists tend to own the entire capital stock) if an independent investment function is introduced into the analysis.[5] It is our view that fiscal activity, with the assumption $t_P < t_W$, could precipitate a movement toward the anti-dual case (at least to guarantee the local stability of the Cambridge equation, as becomes clear in the further numerical example).

It has been observed, in some countries, that the relative shares of national product earned by labour and capital have been quite stable

[4]Steedman (1972) presents a Pasinettian model with government in which, in general, a Meade equilibrium is not possible.

[5]Another solution of the model is the case where no steady-state equilibrium exists.

over many decades. Such an observation suggests the formulation of a production function where factor shares are assumed to be constant, e.g. the Cobb–Douglas production function $Y = K^{\alpha}L^{1-\alpha}$, where α and $1-\alpha$ correspond to the shares of output received by capital and labour respectively (under conditions of perfect competition and profit-maximizing behaviour).

The next step is the local stability analysis of the Cambridge equation solution using the Cobb–Douglas production function. For this purpose we analyse the Jacobian matrix, calculated at the equilibrium point (k^*, k_C^*), noticing that k^* is given by the Cambridge equation:

$$J = \begin{bmatrix} \dfrac{\partial \dot{k}}{\partial k} & \dfrac{\partial \dot{k}}{\partial k_C} \\ \dfrac{\partial \dot{k}_C}{\partial k} & \dfrac{\partial \dot{k}_C}{\partial k_C} \end{bmatrix}$$

We assume that the following inequalities hold:

(i) $\dfrac{s'_{WW}}{s'_C - s'_{WC}} \leq \dfrac{(1-k_C)\alpha}{1-\alpha}$

(ii) $0 < 1 - \dfrac{1}{2\alpha} < k_C < 2 - \dfrac{1}{\alpha}$

It can be shown (see Appendix B) that the sign of each element of the Jacobian matrix is as follows:

$$\frac{\partial \dot{k}}{\partial k} < 0 \qquad \frac{\partial \dot{k}}{\partial k_C} > 0 \qquad \frac{\partial \dot{k}_C}{\partial k} < 0 \qquad \frac{\partial \dot{k}_C}{\partial k_C} < 0$$

In the above case, the determinant of matrix J is positive, $|J| > 0$, and its trace is negative, $\mathrm{Tr}\,J < 0$. These are the conditions for the local stability of the dynamic system formed by equations (3.17) and (3.18) (see, for example, Chiang, 1984).[6] Therefore, we can conclude that the Pasinettian equilibrium is locally stable under conditions (i) and (ii).

Both equilibria can be seen in Fig. 1, which represents the locus of $\dot{k} = 0$ and $\dot{k}_C = 0$ in the space $k \times k_C$. Point A shows the Cambridge equation as one of the steady-state solutions and point B shows the dual equilibrium as the other steady-state solution when the locus $\dot{k}_C = 0$ coincides with the axis k given that $k_C = 0$.

In Fig. 1 point A is the equilibrium point

$$(k^*, k_C^*) = \left(Y_K^{-1} \frac{g}{s'_C}, 1 - \frac{s'_{WW}}{s'_C - s'_{WC}} \frac{1-\alpha}{\alpha} \right)$$

[6]This locally stable equilibrium can be a stable node or a stable focus depending upon the inequality between $(\mathrm{Tr}\,J)^2$ and $4|J|$ (see Chiang, 1984, p. 684).

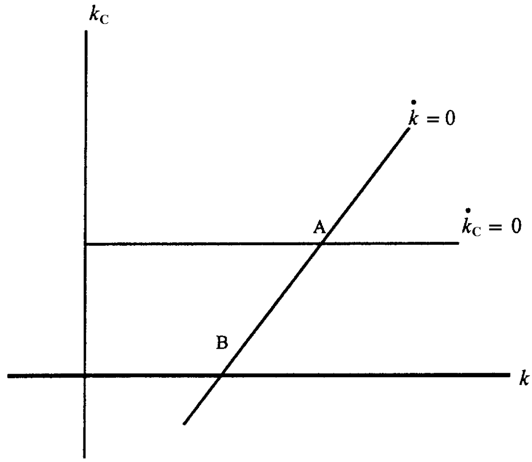

FIG. 1

and point B corresponds to the equilibrium point

$$(k^*, k_C^*) = \left(\left[\frac{s'_{WW}(1-\alpha) + \alpha s'_{WC}}{g} \right]^{1/(1-\alpha)}, 0 \right)$$

A numerical example, based on hypothetical parameters, is given to illustrate a case in which all the required conditions for existence and local stability of the Cambridge equation are satisfied. Let us assume that $s_W = 0.6$, $s_g = 0.7$, $s_C = 0.9$, $t_W = 0.2$, $t_P = 0.1$, $t_i = 0.3$, $\alpha = 0.9$ and $k_C = 0.5$. It follows that $s'_{WW} = 0.7$, $s'_{WC} = 0.6998$ and $s'_C = 0.9076$. These parameters fulfil inequalities (i) and (ii). Considering inequality (i) we have

$$\frac{s'_{WW}}{s'_C - s'_{WC}} = 3.36 < 4.5 = \frac{(1-k_C)\alpha}{1-\alpha}$$

Considering inequality (ii), we have

$$0 < 1 - \frac{1}{2(0.9)} = 0.45 < 0.5 < 0.89 = 2 - \frac{1}{0.9}$$

Moreover, if the natural rate of growth g is 0.02, in equilibrium the rate of profit given by the Cambridge equation ($r = g/s'_C$) is equal to 0.022.

Notice that the high value of the capital share of output ($\alpha = 0.9$) used in the simulation above is too high to be plausible in any real economy. Consequently, we cannot claim that our conditions give any empirical support to the local stability of the Cambridge equation. However, notice that, when $\alpha = 1$, this implies by equation (3.20) that $k_C = 1$, and by equation (3.19) that $r = Y_K = g/s'_C$, which constitute precisely the anti-dual case $(k^*, k_C^*) = (Y_K^{-1}(g/s_C'), 1)$. Therefore, despite the use of unreal values in our numerical example, it shows that the anti-dual case is more likely to occur in order to guarantee the fulfilment of the local stability conditions.

4 Concluding Remarks

It has been shown in this paper that the existence of certain kinds of taxation and government expenditure do not affect the nature of the Cambridge equation, despite the blended framework involving Pasinettian saving behaviour with neoclassical technology. Our model extends the Darity (1981) approach to the case where fiscal policy is a significant feature of the economy. The model developed in terms of a system of differential equations presents the Cambridge equation, the dual equilibrium and the anti-dual equilibrium as possible steady-state solutions.

Local stability of the Cambridge equation was also studied under the assumption that the technology is given by a Cobb–Douglas production function. We have considered the conditions consistent with full-employment growth under perfect competition and profit-maximizing behaviour. A numerical example was presented to show that the conditions examined are not an empty set. Despite the fact that we cannot claim any empirical support to the case studied, since the values are unreal, our results indicate that the anti-dual case is more likely to occur in order to guarantee the local stability of the Cambridge equation.

Appendix A: Basic Notation

G, government expenditure; C, consumption; I, investment
Y, national income; W, wages
P_C, profits accruing to the capitalists; P_W, profits accruing to the workers
s_W, (marginal and average) propensity to save for the workers
s_C, (marginal and average) propensity to save for the capitalists
s_g, proportion of total taxes that is not spent
T, total taxation; t_W, proportional (direct) tax on wages
t_P, proportional (direct) tax on profits
t_i, proportional (indirect) tax on *all* consumption expenditures of all individuals (workers and capitalists) and of the government itself
S_W, workers' saving net of both wages and profits taxes
S_C, capitalists' saving net of profits taxes
S_g, government savings; S, total savings
K, aggregate capital stock; K_W, capital owned by workers
K_C, capital owned by capitalists; L, labour force (total)
N, labour force employed
g, Harrodian 'natural rate' of growth, and population rate of growth
k, capital per head; k_C, capitalists' share of the capital stock
k_W, workers' share of the capital stock; r, rate of profit on capital
A dot above a variable represents the time derivative.

Appendix B: The Relevant Signs for Stability

(i) $$\frac{\partial \dot{k}}{\partial k} = s'_{\mathrm{WW}}[a(k,1) - Y_{\mathrm{K}}] + k(a_{\mathrm{K}} - Y_{\mathrm{KK}}) + s'_{\mathrm{WC}}(1-k_{\mathrm{C}})(Y_{\mathrm{K}} + kY_{\mathrm{KK}}) + s'_{\mathrm{C}}(Y_{\mathrm{K}}k_{\mathrm{C}} + kk_{\mathrm{C}}Y_{\mathrm{KK}}) - g$$

For $k^* \Rightarrow r = Y_{\mathrm{K}}(k^*) = g/s'_{\mathrm{C}} \Rightarrow Y_{\mathrm{K}}s'_{\mathrm{C}} = g$ and supposing the technical possibilities of the economy are represented by the Cobb–Douglas production function $Y = K^{\alpha}L^{1-\alpha}$, with $Y_{\mathrm{K}} = \alpha K^{\alpha-1}L^{1-\alpha}$, $Y_{\mathrm{KK}} = \alpha(\alpha-1)K^{\alpha-2}L^{1-\alpha}$, $a = Y/K = K^{\alpha-1}L^{1-\alpha}$ and $a_{\mathrm{K}} = (\alpha-1)K^{\alpha-2}L^{1-\alpha}$ we can rewrite $\partial\dot{k}/\partial k$ as

$$\frac{\partial \dot{k}}{\partial k} = a\left\{ s'_{\mathrm{WW}} + \alpha(s'_{\mathrm{WC}} - s'_{\mathrm{WW}} - s'_{\mathrm{C}}) + \alpha k_{\mathrm{C}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}}) + \frac{\alpha-1}{L}[s'_{\mathrm{WW}} + \alpha(s'_{\mathrm{WC}} - s'_{\mathrm{WW}}) + \alpha k_{\mathrm{C}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}})]\right\}$$

The sign of $\partial\dot{k}/\partial k$ is negative if

$$s'_{\mathrm{WW}} + \alpha(s'_{\mathrm{WC}} - s'_{\mathrm{WW}} - s'_{\mathrm{C}}) + \alpha k_{\mathrm{C}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}}) < 0$$

This is satisfied if and only if

$$\frac{s'_{\mathrm{WW}}}{s'_{\mathrm{C}} - s'_{\mathrm{WC}}} < \frac{\alpha(1-k_{\mathrm{C}})}{1-\alpha}$$

Since $s'_{\mathrm{WW}} > s'_{\mathrm{C}} - s'_{\mathrm{WC}} \Rightarrow s'_{\mathrm{WW}}/(s'_{\mathrm{C}} - s'_{\mathrm{WC}}) > 1$, hence $\alpha(1-k_{\mathrm{C}})/(1-\alpha) > 1$ is a necessary condition if $k_{\mathrm{C}} < 2 - 1/\alpha$.

(ii) The sign of $\partial\dot{k}/\partial k_{\mathrm{C}}$ is positive since $kY_{\mathrm{K}} > 0$ and $s'_{\mathrm{C}} - s'_{\mathrm{WC}} > 0$.

(iii) The sign of $\partial\dot{k}_{\mathrm{C}}/\partial k$ is negative if

$$(1-k_{\mathrm{C}})Y_{\mathrm{KK}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}}) - s'_{\mathrm{WW}}(a_{\mathrm{K}} - Y_{\mathrm{KK}}) < 0$$

$$\Rightarrow (1-k_{\mathrm{C}})Y_{\mathrm{KK}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}}) < (a_{\mathrm{K}} - Y_{\mathrm{KK}})s'_{\mathrm{WW}}$$

this being true if and only if $k_{\mathrm{C}} < 2 - 1/\alpha$.

(iv) Finally, the sign of $\partial\dot{k}_{\mathrm{C}}/\partial k_{\mathrm{C}}$ is negative when

$$Y_{\mathrm{K}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}})(1-2k_{\mathrm{C}}) - s'_{\mathrm{WW}}(a - Y_{\mathrm{K}}) < 0$$

$$\Rightarrow Y_{\mathrm{K}}(s'_{\mathrm{C}} - s'_{\mathrm{WC}})(1-2k_{\mathrm{C}}) < s'_{\mathrm{WW}}(a - Y_{\mathrm{K}})$$

Since $s'_{\mathrm{WW}} > s'_{\mathrm{C}} - s'_{\mathrm{WC}}$, hence $Y_{\mathrm{K}}(1-2k_{\mathrm{C}}) < a - Y_{\mathrm{K}}$; it is true if and only if $k_{\mathrm{C}} > 1 - 1/2\alpha$.

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