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Investment, Credit, and Endogenous Cycles

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This paper presents a general-equilibrium dynamic Ramsey-type model that can generate endogenous cycle. We assume two different representative agents, borrowers and lenders, and financial intermediaries with inside and outside money. We investigate under which conditions this model presents a cyclical relationship between capital and loans. The sources of endogenous fluctuations in this model come from a credit restriction in the representative-borrower problem.

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1 Introduction

In the current literature there are two different approaches to dealing with the relationship between money and cycles. The first considers exogenous shocks according to the real-business-cycle theory. These shocks are studied in four different ways[1]: (i) shocks in the financial industry (King and Plosser, 1984); (ii) cash injections in the cash-in-advance restriction (Cooley and Hansen, 1989); (iii) cash injection in the financial intermediaries and different timing between agents' decisions and the occurrence of shocks (Christiano, 1994), and (iv) monetary shocks in imperfect financial markets (Scheinkman and Weiss, 1986).

The second approach assumes that economic fluctuations arise endogenously. The endogenous fluctuations are studied in perfect-competition models and in models exploring some market failures. The first type of models stress the role of agents' high degree of impatience, strong income effects, and low substitutability between capital and labor in technology as sources of endogenous cycles (see Reichlin, 1998).

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[1] For a survey on real business cycles and money, see Van Els (1995).

The second type of models, on the other hand, depart from the Marxist and Keynesian tradition and explore the instability of decentralized decision-making in market economies (Foley, 1992) and some sources of market imperfection (e.g., Asada and Semmler, 1995).

One of the main differences between these two approaches are the tools used to generate cycles. Technically the models in the real-business-cycle tradition are developed from micro-founded dynamic models to be numerically tested by calibration and have some empirical support (Kydland and Prescott, 1996). In the endogenous business cycle the models are mainly theoretical and use qualitative differential-equation theory to solve them. The application of the Poincaré–Bendixson (e.g., Jarsulic, 1988; Gonzalez-Calvet and Sanchez-Choliz, 1994) and the Hopf bifurcation (e.g., Kiefer, 1996) theorems are common tools used to create limit cycles in these models.

Our paper presents a general-equilibrium dynamic Ramsey-type model that can generate endogenous cycles. We assume two different representative agents, borrowers and lenders, as in Mossetti (1990), and financial intermediaries with inside and outside money as in Ohkusa (1993). Using the method developed by Feichtinger et al. (1994) we investigate under which conditions this model presents a cyclical relationship between investment and loans. The sources of endogenous fluctuations in this model come from a credit restriction in the representative-borrower problem. The paper is structured as follows. Section 2 presents the basic model. In Sect. 3 we show how endogenous cycle can be achieved and in Sect. 4 we conclude.

2 The Model

The model has three types of agents: financial intermediaries, lenders, and borrowers. There are two types of money, inside and outside money. Outside money enters into the economy through cash injections from the government to the financial intermediaries. There are three markets in the economy, money, credit, and goods markets. In equilibrium, the money market defines the optimum quantity of outside money. The credit market defines the optimal values of deposits, loans, and the nominal interest rates for deposits and loans. Finally, the goods market determines the optimal quantity of capital and consumption of each agent.

We assume that the representative lender derives utility from consumption, c, and money holdings m. It is assumed that money enters in the utility function.[2] Her savings are given by the difference between the net revenue of deposits in banks, $(\phi - \pi)D$ (where D is the real amount of deposits, ϕ the nominal interest rate paid by deposits, and π the inflation rate), and consumption and inflationary tax, πm. The savings are allocated in bank deposits and money. The problem of the representative lender is the following:

$$\max_{c,m} \int_0^\infty U(c,m)\mathrm{e}^{-rt}\,\mathrm{d}t, \quad \text{s.t. } \dot{D} + \dot{m} = (\phi - \pi)D - c - \pi m\ , \quad (1)$$

where r is the rate of time preference. Notice that the representative lender allocates in her portfolio inside (D) and outside (m) money.[3]

The representative borrower maximizes a flow of discounted consumption (c') subject to a budget constraint and to a credit restriction in which part of her loans (sL), if s is defined in the interval $(0, 1]$, finances her investment ($\mathrm{d}k/\mathrm{d}t$), adjustment costs $[C(\mathrm{d}k/\mathrm{d}t, k)]$, and consumption decisions. In the case in which $s > 1$, the credit restriction says that part of her expenditures is financed by loans, so the rest can be financed by equity finance.[4] Her budget constraint corresponds to the allocation of the production $[f(k)]$ and a new loan ($\mathrm{d}L/\mathrm{d}t$) in consumption, investment, adjustment costs, and the payment of her debt iL, where i is the interest rate paid by borrowers. Inflation represents a new source of income to the borrowers, since the real value of their debt decreases with it. We assume, as Mossetti (1990), that the representative borrower does not hold outside money. The problem of the representative borrower is the following:

$$\begin{aligned} &\max_{c'} \int_0^\infty V(c')\mathrm{e}^{-rt}\,\mathrm{d}t, \\ &\text{s.t. } \dot{k} - \dot{L} = f(k) - c' - (i - \pi)L - \delta k - C(\dot{k}, k)\ , \\ &\qquad sL \geq c' + \dot{k} + C(\dot{k}, k)\ , \end{aligned} \quad (2)$$

where L is the real amount of loans, δ the depreciation rate, and i the

2 This is known as the most general way to introduce money in growth models (see Feenstra, 1986).

3 Note that m appears as state and control variable in problem (1). This problem can be solved (see, e.g., Blanchard and Fischer, 1989) by defining a new state variable, B, as $B = D + m$. The dynamic restriction is rewritten as: $\dot{B} = (\phi - \pi)B - \phi m - c$. Then we can consider m as a control variable.

4 In imperfect capital markets the investment decisions are primarily financed by retaining profits and, when exhausted, by debt, and last, by equity finance (see Fazzari et al., 1988).

rate of interest paid for the loan.[5] Notice that we are assuming the same rate of time preference for both agents, since the assumption of different rates of time preference does not bring any additional insight in our model.

We can rewrite problem (2) by making

$$\dot{k} = I \ . \tag{3}$$

This transformation makes the borrower problem look like a firm's problem restricted by the availability of credit; using (3) in the second restriction of (2) yields

$$c' = sL - I - C(I, k) \ , \tag{4}$$

since the inequality in this restriction reduces to an equality due to the insatiability of consumers. Defining the adjustment costs as an increasing function of the ratio of investment to capital yields[6]

$$C(I, k) = \alpha I k^{-1} \ . \tag{5}$$

Inserting (3), (4), and (5) into problem (2) we get[7]:

$$\begin{aligned} &\max_{I} \int_0^\infty V(sL - I(1 + \alpha k^{-1}))\mathrm{e}^{-rt}\,\mathrm{d}t, \\ &\text{s.t. } \dot{L} = (s + i - \pi)L + \delta k - f(k), \ \ \dot{k} = I \ . \end{aligned} \tag{6}$$

To close the model we have two equilibrium conditions that hold in the steady state for the financial intermediaries. They lend deposits $(1 - \theta)D$, where θ is the preparation rate, plus nonstochastic cash injection $Z(m)$ from the monetary authority,[8] which is the way outside money enters into the economy (see Christiano, 1994; Ohkusa, 1993):

$$L = Z(m) + (1 - \theta)D \ . \tag{7}$$

5 Notice that the same rate of time preference is assumed for both agents. Different rates of time preference would generate new arbitrage conditions in equilibrium, which are not the focus in the present paper.

6 For the case of disinvestment, we can assume that Eq. (5) is a module function, which is a weakly convex function (Das, 1991).

7 Notice that problem (6) is very similar, in its restrictions, to the model analyzed by Asada and Semmler (1995).

8 The function $Z(m)$ can be associated to the seignorage from money.

We assume that all deposits and loans clear up:

$$L = D \ . \tag{8}$$

Solving problems (1) and (6), where we assume a CRRA utility function, in the case of the representative borrower we have[9]: $V(\cdot) = (sL - I(1+\alpha k^{-1}))^{1-\sigma}/(1-\sigma)$. Taking the equilibrium conditions for the financial sector, we obtain a system of nine equations to nine unknowns: I, L, D, ϕ, i, k, q, c, m, where q is the costate variable associated with k in problem (6):

$$I = 0 \ , \tag{9}$$

$$\phi = r + \pi \ , \tag{10}$$

$$L = D \ , \tag{11}$$

$$L = Z(m) + (1-\theta)D \ , \tag{12}$$

$$c + \pi m = (\phi - \pi)D \ , \tag{13}$$

$$U_m = \phi U_c \ , \tag{14}$$

$$f(k) = L(s + i - \pi) + \delta k \ , \tag{15}$$

$$L = (1 + \alpha k^{-1})^{1/\sigma} q^{-1/\sigma} s^{-1} \ , \tag{16}$$

$$(\delta - f_k)(sL)^{-\sigma} s = rq(r - i - s + \pi) \ . \tag{17}$$

This system is block recursive, Eq. (9) determines optimal I and Eq. (10) determines optimal ϕ. From Eqs. (11)–(17) the optimal values of L, D, m, c, i, k, and q are simultaneously determined. Therefore, the markets for credit, money, and goods clear at the same time.

3 The Cycle

The endogenous cycle arises from the analysis of problem (6). Following Feichtinger et al. (1994) it is enough to show that the signs of

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9 Despite the use of a linear adjustment-cost function, the CRRA yields the necessary conditions for an optimum as in Lucas (1967).

$$\det J = \begin{vmatrix} \frac{\partial \dot{L}}{\partial L} & \frac{\partial \dot{L}}{\partial k} & \frac{\partial \dot{L}}{\partial \lambda} & \frac{\partial \dot{L}}{\partial q} \\ \frac{\partial \dot{k}}{\partial L} & \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial \lambda} & \frac{\partial \dot{k}}{\partial q} \\ \frac{\partial \dot{\lambda}}{\partial L} & \frac{\partial \dot{\lambda}}{\partial k} & \frac{\partial \dot{\lambda}}{\partial \lambda} & \frac{\partial \dot{\lambda}}{\partial q} \\ \frac{\partial \dot{q}}{\partial L} & \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial \lambda} & \frac{\partial \dot{q}}{\partial q} \end{vmatrix} , \tag{18}$$

$$\Omega = \begin{vmatrix} \frac{\partial \dot{L}}{\partial L} & \frac{\partial \dot{L}}{\partial \lambda} \\ \frac{\partial \dot{\lambda}}{\partial L} & \frac{\partial \dot{\lambda}}{\partial \lambda} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{k}}{\partial k} & \frac{\partial \dot{k}}{\partial q} \\ \frac{\partial \dot{q}}{\partial k} & \frac{\partial \dot{q}}{\partial q} \end{vmatrix} + 2 \begin{vmatrix} \frac{\partial \dot{L}}{\partial k} & \frac{\partial \dot{L}}{\partial q} \\ \frac{\partial \dot{\lambda}}{\partial k} & \frac{\partial \dot{\lambda}}{\partial q} \end{vmatrix} . \tag{19}$$

and a condition to determine the value of the bifurcation parameter:

$$\det J = (\Omega/2)^2 + r^2(\Omega/2) \tag{20}$$

are positive when calculated with the optimal solutions of L, k, λ, and q in order that matrix J has a pair of purely imaginary eigenvalues. The value of λ is determined from the first-order conditions to problem (6), where λ is the costate variable associated with state variable L.

From the first-order conditions of problem (6) we have:

$$I = [sL - (1 + \alpha k^{-1})^{1/\sigma} q^{-1/\sigma}](1 + \alpha k^{-1})^{-1} , \tag{21}$$

$$\dot{\lambda} = \lambda(r - s - i + \pi) - [sL - I(1 + \alpha k^{-1})]^{-\sigma} s , \tag{22}$$

$$\dot{q} = rq - \lambda(\delta - f_k) - [sL - I(1 + \alpha k^{-1})]^{-\sigma} (I\alpha k^{-2}) \tag{23}$$

from (3) and (21) in the steady state we obtain Eqs. (9) and (16). As in the steady state (17) comes from the equality of (22) and (23), the optimal value of λ can be determined by (22) or (23).

Consider the following two inequalities:

$$\begin{aligned} &(f_k - \delta)^2 (sL)^{-\sigma-1} s^2 I_q \sigma > \\ &\quad -(r - s - i + \pi) r (I_k(s + i - \pi) + (f_k - \delta) I_L) \\ &\quad + \lambda f_{kk}(r - s - i + \pi)(s + i - \pi) I_q , \end{aligned} \tag{24}$$

$$I_k r + 2I_L(f_k - \delta) > -(r - s - i + \pi)(s + i - \pi) + \lambda f_{kk} I_q , \tag{25}$$

where I_x denotes the partial derivative of I in relation to $x = q, L, k$.

If inequalities (24) and (25) calculated with the optimal solutions of the model (9)–(17) are preserved and the bifurcation parameter α calculated from (20) is positive for the same optimal solutions, then there is a limit cycle, by the Hopf bifurcation theorem, between loans (L) and capital (k) in the economy described by problems (1), (6), and the equilibrium conditions (7), (8).

We can see this result noticing that inequality (24) yields $\det J > 0$, and inequality (25) yields $\Omega > 0$. And if by (20) the bifurcation parameter is positive, these three conditions are necessary such that matrix J possesses a pair of purely imaginary eigenvalues. These fulfill the conditions for the existence of a limit cycle by the Hopf bifurcation theorem (Feichtinger et al., 1994).

The above result holds true for specific values of parameters, $A = 0.25$, $r = 0.045$, $\pi = 0.06$, $s = 0.02$, $\theta = 0.6$, $\sigma = 1.1$, $\delta = 0.10$, $\beta = 0.9$. By Cobb–Douglas production and utility functions, $f(k) = Ak^{\beta}$, $U(c, m) = c^{2/3}m^{1/3}$, and the following cash injection from the monetary authority $Z(m) = 1 + m$, we have positive signs for $\det J$ and Ω and $\alpha = 0.294998$. Therefore, a limit cycle between credit and capital – not necessarily a stable one – exists in our model.[10] However, as in Wirl (1994), our interest is just to show under which conditions the existence of a limit cycle is possible, not to discuss its stability.

The cyclical behavior has two sources in this model. The first results from the positive externality due to L in the objective functional of problem (6), which allows k to differ from the optimal k from the modified golden rule. The modified golden rule can be obtained by solving problem (2) after dropping the adjustment costs and the credit restrictions from it.

The second source of cycles is related to the penalization of changes in the investment. The main mechanism to guarantee this penalty is given by the impact of k on I. When the impact is positive this implies that the penalty is high. We can see this by inequality (25), that demands a positive effect of k on I to be verified. Notice that this effect is only possible given the dependence of the adjustment costs on k. It is easy to see that an increase in k decreases adjustment costs and it stimulates the investment.

It is important to notice that both sources of cycles in our model

10 The values of the endogenous variables for this configuration of parameters are: $L = 2.3076923$, $m = 0.3846153$, $c = 0.0807692$, $k = 1.4424173$, $q = 35.496377$, $i = 0.1281365$. Using those results in the model we found: $\det J = 6.506 \cdot 10^{-8}$ and $\Omega = 0.00006327$. It is important to notice that the system may present multiple solutions.

come from the credit restriction in the problem of the representative borrower.

4 Conclusions

Our general-equilibrium setup with heterogeneous representative agents in the Ramsey framework gives us the mechanics of the endogenous cycle between capital and loans. In the problem of the representative borrower the presence of a positive externality given by the existence of loans in the objective functional, and a penalty for changes in the investment associated with capital in the adjustment-cost function are the main features of our model which suffice to generate an endogenous cycle for specific values of the parameters. These two features come from the inclusion of a credit restriction in the borrower problem. The cycle is still consistent with optimal choices from the representative lender and from the financial intermediaries, and it shows how in a general-equilibrium setup it is possible to have a cyclical pattern between capital and loans.

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