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Supervision and effort in an intertemporal efficiency wage model: the role of the Solow condition

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Abstract

The Solow condition is examined in an intertemporal model that blends the shirking and the turnover models of efficiency wages with managerial supervision. It is shown that the Solow condition does not hold when shirking and turnover costs are considered. The Solow condition can be a possible outcome when managerial productivity offsets shirking and turnover costs. © 2000 Elsevier Science S.A. All rights reserved.

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1. Introduction

Efficiency wage models have been developed to explain involuntary unemployment. Firms may benefit from paying their workers more than the market clearing wage, thus generating involuntary unemployment. Among the efficiency wage models, the shirking and the turnover cost models are the most well known and cited. According to the shirking model (e.g., Shapiro and Stiglitz, 1984), firms pay efficiency wages in order to reduce workers' shirking. High wages make the workers fear to lose their jobs if they are caught shirking. In the turnover cost model (e.g., Salop, 1979), workers who quit have to be replaced, which makes the firms incur search, recruitment and training costs. So firms have an incentive to minimize these costs. One such device is to set the wage at a level which inhibits turnover.

One of the standard results of the efficient wage models is due to Solow (1979). The Solow condition, as it became known, states that an optimizing firm sets its wage at the level which the

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elasticity of work effort with respect to the real wage is unity. However, this result came under attack. Akerlof and Yellen (1986) point out that an effort-wage elasticity of unity is quite high. If this elasticity is never that high, “then there cannot be an equilibrium with unemployment in an efficiency wage model” (Akerlof and Yellen, 1986, p. 14). As the Solow model does not contemplate shirking, workers who do not work hard do not waste firm’s output since additional workers can be hired. If additional costs related to low-effort labor are taken into consideration, this will result in an equilibrium effort-wage elasticity lower than unity.

On the other hand, efficiency wages have often been used to explain the pattern of interindustry wages (e.g. Krueger and Summers, 1988). One empirical regularity that they sought to explain is that industries with greater productivity per employee pay their workers more (e.g. Katz and Summers, 1989). Mehta (1998) proposed a model in which supervisors monitor and coordinate their subordinates. However managers are constrained to make tradeoffs in these activities. Therefore “if the productivity of supervisors increases, then other things being equal, they will want to coordinate more and monitor less. Consequently, to maintain effort, they will pay more. Similarly, an increase in the productivity of workers makes supervisors want to employ more of them. Other things being equal, they will now want to monitor each worker less and pay more” (Mehta, 1998, p. 153).

This paper presents a model that combines the shirking and turnover costs models with managerial supervision in an intertemporal optimizing framework. The model encompasses Marti (1997) and Mehta (1998) analysis into an intertemporal optimization framework developed along the lines of Lin and Lai (1994). The results show that when managerial productivity is not considered, and shirking and turnover costs are taken into account, the Solow condition is not valid. Furthermore, when managerial productivity is considered and offsets shirking and turnover costs, then the Solow condition can be a possible outcome.

2. The Model

Following Mehta (1998), the representative firm has a production function that depends on the product of managers and workers:

$$y = S(1 - Mg(s))sN + \Omega \frac{\delta(s)}{s} e(w)sN \quad (1)$$

the first term on the right-hand side (RHS) stands for the net contribution that supervisors make to directly increasing output. Where S is the productivity of supervisors, M is the difficult of monitoring, $g(s)$ is the extensive monitoring (assumed to be a convex function, $g' > 0$, $g'' > 0$), and s is the span of control of each supervisor. There are N identical teams with s workers, so the total number of workers is $n = Ns$. The second term in the RHS is the net output of workers and depends on the productivity of workers (Ω). The function $\delta(s)$ indicates how output of the s workers in the productive unit, who work for one supervisor, varies with s ($\delta_s > 0$; $\delta_{ss} < 0$). Workers’ effort (e) is assumed to be an increasing function of the relative wage (w), $e_w > 0$. The relative wage is defined as the difference between the wage paid by the firm (w_F) and the reservation wage (w_R): $w = w_F - w_R$. In the above production function it is implicitly assumed that the probability of detection of shirking is equal to one.

It is also assumed that the representative firm is a monopolistic competitor in the goods market. The

firm chooses three variables to maximize the discounted profit over an infinite horizon. It uses the span of control (s), the number of hired workers (h) and the relative wage (w) as control variables in the problem below:

$$\text{Max}_{h,w,s} \int_0^{\infty} [p(y)y - wn - c(N) - \tau(h) - \sigma x] \exp(-rt) dt \quad (2)$$

$$\text{s.t. } \dot{n} = h - q(w)n \quad (3)$$

The term in the brackets in Eq. (2) captures the training costs, $\tau(h), \tau_h > 0$, when $h > 0$, and the search costs (σx), where x is the search effort to hire new workers and σ is the unit cost of search. In order to hire, the firm must expend search effort, where $\theta = h/x$, is the number of hires per unit of search. θ is a measure of search effectiveness of the firm. θ is assumed to be an increasing function of the relative wage ($\theta = \theta(w), \theta_w > 0$). The function $c(N)$ is a convex cost of coordination function. Eq. (3) describes the time variation of employed workers. It depends on the difference between the number of hired workers and the workers who decide to quit from the firm. Where $q(w)$ is the quit rate, which is assumed to be a decreasing function of the relative wage ($q_w < 0$). If managers are not productive ($S = 0$) and there are no coordination costs ($c(N) = 0$), and the quit rate (q) and search effectiveness (θ) are independent of the relative wage, we have the shirking model; and if ($S = 0, c(N) = 0$), and the effort of workers is independent of the relative wage, we have a version of the turnover cost model (Marti, 1997). When q, θ and e are independent of the relative wage and S and $c(N)$ are different from zero, we have a dynamic version of Mehta (1998) model. Therefore, the model above combines the shirking and the turnover cost model with the managerial supervision model in an intertemporal optimization framework.

The current value Hamiltonian function, H , is written as:

$$H = p(y)y - wn - c\left(\frac{n}{s}\right) - \tau(h) - \frac{\sigma h}{\theta(w)} + \lambda[h - q(w)n]$$

Introducing (1) into the Hamiltonian, the first order conditions of this problem are:

$$\lambda = \tau_h + \frac{\sigma}{\theta} \quad (4)$$

$$y_s p(y)(1 + \psi) = c_n \frac{n}{s^2} = 0 \quad (5)$$

$$(1 + \psi)p(y)y_w = n(1 + \lambda q_w) - \frac{\sigma h \theta_w}{n \theta^2} \quad (6)$$

$$\dot{\lambda} = \lambda(r + q) - \left[(1 + \psi)p(y)y_n - w - \frac{c_n}{s} \right] \quad (7)$$

where

$$\psi = \frac{dp}{dy} \frac{y}{p}$$

Considering the steady state:

$$\dot{n} = 0 \Rightarrow q(w)n = h$$

and by using (4) and (6) in (7) when $\dot{\lambda} = 0$, we have:

$$e_w \frac{w}{e} = \left[\frac{1 + (\tau_h + \sigma/\theta)q_w - \sigma q \theta_w / \theta^2}{1 + c_n / w s + (\tau_h + \sigma/\theta)(r + q) / w} \right] \left[1 + \frac{S(1 - Mg(s))}{\Omega(\delta(s)/s)e} \right] \quad (8)$$

From (8) we can discuss the Solow condition.

3. The Solow condition

Consider first a model without managerial supervision, i.e., $S = 0$ and $c(N) = c_n = 0$. Eq. (8) reduces to

$$e_w \frac{w}{e} = \frac{1 + q_w(\tau_h + \sigma/\theta) - [(\sigma q \theta_w) / \theta^2]}{\{1 + (\tau_h + \sigma/\theta)[(r + q) / w]\}} \quad (9)$$

As it is assumed that the number of hired workers, h , is positive, then: $\tau_h > 0$; and taking into consideration that: $\theta_w > 0$, and $q_w < 0$, it follows that the numerator is less than unity and the denominator is greater than unity in Eq. (9). Therefore the wage-elasticity of effort is less than unity:

$$e_w \frac{w}{e} < 1$$

From (9), three special situations can be analysed. First, if the voluntary quitting rate does not vary with the relative wage ($q_w = 0$), Eq. (9) reduces to:

$$e_w \frac{w}{e} = \frac{1 - [(\sigma q \theta_w) / \theta^2]}{1 + (\tau_h + \sigma/\theta)[(r + q) / w]} \quad (10)$$

which is still less than unity.

The second case is related to the situation in which the search effectiveness of the firm does not depend on the relative wage ($\theta_w = 0$):

$$e_w \frac{w}{e} = \frac{1 + q_w(c_h + \sigma/\theta)}{1 + (\tau_h + \sigma/\theta)[(r + q) / w]} \quad (11)$$

which is also less than unity.

As seen above, our optimization problem reduces to a shirking model when $\theta_w = 0$, and $q_w = 0$. Then combining (11) and (10) yields:

$$e_w \frac{w}{e} = \frac{1}{1 + (\tau_h + \sigma/\theta)[(r + q) / w]} \quad (12)$$

so, as in Akerlof and Yellen (1986), when shirking is taken into consideration, the effort-wage elasticity is less than unity, and the Solow conditions do not hold.

The third situation occurs when the training costs are absent ($\tau(h) = \tau_h = 0$). From Eq. (9) we have:

$$e_w \frac{w}{e} = \frac{1 + q_w \sigma / \theta - [(\sigma q \theta_w) / \theta^2]}{1 + \sigma / \theta [(r + q) / w]} \quad (13)$$

which is less than unity.

The three situations above show that the Solow condition is invalid when shirking and turnover costs are taken into account.

Returning to Eq. (8), the Solow condition can be verified if and only if:

$$e_w \frac{w}{e} = 1 \Leftrightarrow \left[\frac{S(1 - Mg(s))}{\Omega(\delta(s)/s)e} \right] = \left[\frac{1 + c_n/ws + (\tau_h + \sigma/\theta)(r + q)/w}{1 + (\tau_h + \sigma/\theta)q_w - \sigma q \theta_w / \theta^2} \right] - 1 > 0 \quad (14)$$

condition (14) is the heart of the paper. On the one hand it asserts that the Solow condition depends critically on the rate of productivity of managers and workers (S/Ω). If this rate is high, *ceteris paribus*, the Solow condition can be verified. That is, if the managers have high productivity vis-à-vis workers, they can overcome the problems generated by shirking and turnover costs. On the other hand, giving (S/Ω), and considering the span of control of each supervisor (s), the extensive monitoring ($g(s)$), and output of a team under one supervisor ($\delta(s)$), it is easy to see that an increase in the variable s is associated with a decrease in the left-hand side of (14). That means that an increase in the number of people to be monitored by the manager decreases his efficiency. Both results are quite appealing since managers are employed, among other things, to reduce shirking and turnover costs. Therefore, when managerial productivity is considered, it can offset the former effect of shirking and turnovers costs in order to guarantee that the Solow condition holds¹.

4. Concluding remarks

This paper blends the shirking and the turnover models of efficiency wage with managerial supervision into an intertemporal optimization framework. It is shown that the Solow condition does not hold when shirking and turnover costs are considered. The Solow condition can be a possible outcome when managerial productivity offsets shirking and turnover costs.

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¹Notice that these considerations hold true not only for the Solow condition. One can show the elasticity of work effort with respect to the real wage is greater than one as well.

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